Vol. XXXI, B, No: 1 April, 1961

JOURNAL

OF

THE MADRAS UNIVERSITY

"Mode of Citation: *J. Madras Univ. B*"

Editor

PROFESSOR T. S. SADASIVAN, D.Sc., F.N.I., F.A.Sc.

Editorial Board

PROF. C. P. GNANAMUTHU, M.A., D.Sc., F.Z.S,

PROF. P. R. JAGAPATHI NAIDU, M.Sc., PH.D.

PROF. V. S. KRISHNAN, M.A., D.Sc.

PROF. G. S. LADDHA, M.S. CH.E., PH.D.

PROF. D. V. RAJALAKSHMAN, M.A., M.Sc., PH.D.

PROF. G. N. RAMACHANDRAN, M.A., PH.D., D.Sc.

Issued on 12-7-1961

PUBLISHED BY THE UNIVERSITY
MADRAS

Printed and Published by S. Ramaswami, at G. S. Press,
21, Narasingapuram Street, Mount Road, Madras.

CONTENTS

BIONOMICS AND CONTROL OF *PERINA NUDA* FABR., A PEST ON *FICUS* SP. IN SOUTH INDIA
By S. VENUGOPAL, M.Sc. .. 1

RELATIONS BETWEEN SUMS OF SINGULAR SERIES ASSOCIATED WITH CERTAIN QUADRATIC FORMS
By K. ANANDA RAU .. 7

STUDIES ON INDIAN CRYPTONEMIALES—I: *GRATELOUPIA* C.A. AG.
By M. S. BALAKRISHNAN .. 11

A CONTRIBUTION TO THE EMBRYOLOGY OF *EPITHEMA CARNOSUM*
By D. PADMANABHAN .. 37

EMBRYOGENESIS IN *DORSTENIA INDICA*
By D. PADMANABHAN .. 47

STUDIES ON SEEDS WITH RUMINATE ENDOSPERM: 1. MORPHOLOGY OF RUMINATING TISSUE IN *MYRISTICA FRAGRANS*
By K. PERIASAMY .. 53

FURTHER ADDITIONS TO THE LIST OF *MONONCHUS* FROM MADRAS CITY
By S. KANNAN .. 59

SOIL NEMATODES OF THE GENERA *MONONCHUS* AND *MONONCHULUS*
By S. KANNAN .. 63

NEMATODES OF THE GENUS *MONONCHUS* FROM MADRAS CITY
By S. KANNAN .. 69

ON THE NET OF CONICS WITH A COMMON SELF-POLAR TRIANGLE: (PART I)
By B. R. SRINIVASAN .. 75

ON SOME PERMUTABLE PROCESSES IN SEMI-GROUPS
By N. SANKARAN .. 97

J. Madras Univ., B. XXXI, No. 1, Pages 1-5, 1961.,

Bionomics and Control of *Perina nuda* Fabr., a Pest on *Ficus* sp. in South India

BY

S. VENUGOPAL, M.Sc.,

Madras Agricultural Department

(Received for publication on 6-11-1959)

ABSTRACT

The life cycle of *Perina nuda* is described. The full life cycle takes about 28 to 34 days. All the stages are described in detail. This moth is a pest on various trees. Control measures are suggested.

Introduction :

Ficus carica is one of the important fruit plants occasionally attacked by different species of caterpillars which assume some importance when they are in large numbers. Among them *Perina nuda* Fabr. assumes the status of a major pest in Coimbatore and during the periods of abundance defoliate the trees, leaving only the mid-rib and larger veins of the leaves with remnants of leaf tissue. It has a wide distribution throughout India, Ceylon and China as reported by Hampson (1892).

Alternate Host Plants:

It is commonly observed to feed on the leaves of *Mangifera indica* (Barlow, 1900), *Ficus indica* (Fletcher, 1919), *Ficus religiosa* and *Artocarpus integrifolia* (Lefroy, 1909). It was recorded by Ramachandra Rao on Figs in 1917. The following is an account on the larval features, bionomics and control aspect of the pest.

Life-history :

The adults begin to lay eggs in 3 to 5 days after emergence. Each female is capable of laying about 280 eggs and they are laid in clusters on the undersurface of the leaves.

Eggs :

The individual egg is brick-red in colour, rounded and flat with a circular depression at the tip. Measures about 1 mm. in length. The egg period lasts from 5 to 6 days.

Larva :

The newly hatched larva is 3-5 mm. in length and orange yellow in colour. Head and prothorax are bigger than the body segments. Head shield is dark brown. Prothorax is yellowish with light orange coloured processes. II and III thoracic as well as 3rd, 8th, 9th and 10th abdominal segments are pale white or yellow in colour. There are five pairs of prolegs which are slender. The long lateral tubercles on 1st and 2nd segments are blackish with longer racemose hairs.

Second instar :

The larva after first moult measures 100 mm. Head portion is big and black in colour with reddish brown prothoracic processes. The thoracic portion is light greenish with ashy white irregular bands on I and III thoracic segments. The first thoracic segment bears black band across and having reddish brown dorsal paired tubercles and lateral processes with a white line traversing across and separating the head shield. Paired tufts black or grey black are noted on 1st and 2nd abdominal segments. 3rd and 6th abdominal segments are black laterally with median white dorsel area intercepted longitudinally by thin streaks and intersegmentally by light brown patches. The 7th abdominal segment bears a pink coloured protuberance and the 8th segment has a reddish brown pair of tubercles. The ninth segment is dirty yellow dorsally. Hairs on sides are longer than on body surface.

Third instar :

The larva attains a length of 12 mm. and the general colouration is black with white median longitudinal band and with long white lateral hairs. The head is distinctly black. The 6th abdominal segment has a white globular and glandular protuberance. The 7th segment bears a similar yellow protuberance.

Fourth instar :

The larva measures 16 mm. in length and the general appearance is more or less similar to the previous stage.

Fifth instar :

The larva after this moult measures 20 mm. in length and shows no distinct difference from the previous stage. Head is olive brown in colour. On the first thoracic segment 3 dull red coloured tufts of hairs are seen on the lateral surfaces.

Sixth instar:

The full grown larva is 25 mm. long. The head is ashy grey in colour and slightly bigger than body segments. The prothoracic segment is fairly big, ashy white in colour, with median paired turbercles and with red lateral processes. Hairs are white on segments II and III with a median dorsal band and lateral black parch on yellowish white tubercles. The abdominal segments appear whitish ventrally and ventrolaterally with a median dusky grey dorsal area. There are erect tufts of hairs dusky grey in colour dorsally on segments 1 and 2. A distinct fine black streak is noted on the mid-dorsal dusky grey band. Dorsal tubercles are located on either side of dusky grey band like eye spots with black base and ashy white centre. Hairs on the tubercles are dark brown and smaller. Anal segment is whitish. The characteristic glands on 6 and 7th are yellowish white and deep red respectively and appear as conical protuberance. Numerous white hairs are visible on head shield. The setal arrangement is characterised by the presence of verrucal and dense group of hairs on the dorsal side of 1st and 2nd segments. The crochets on prolegs are uniordinal and are borne on slender fairly long prolegs bearing a few secondary setae. The larval period lasts for 18 to 22 days.

Pupa:

Pupa is brownish green in colour with no definite cocoon with supporting silken strands. The pupa is hairy and brightly coloured on the dorsal aspect and pale yellowish white on the ventral portion. The pupal period varies from 5 to 6 days.

Moth:

The following description has been adopted from "The Lepidoptera of Ceylon" (Vol. II) by F. Moore (1883).

Male:

Fore-wing transparent, except at base, which is obliquely covered with pale ochreous brown scales, hind wing pale ochreous brown, darkest towards anal angle; a transparent spot at the apex; thorax pale ochreous brown, abdomen blackish, with narrow whitish segmental bands, anal tuft reddish ochreous; antennae brown.

Female :

Dull ochreous-white, the base of posterior border sparsely irrorated with minute black scales. Body dull ochreous white; head palpi, legs and anal tuft pale ochreous. Expanse, male 1⅜, 1⅝ inch. Generally moths begin to emerge in October. The females are much longer and have cream-white wings and body. On emergence the females are sluggish and dull in disposition and inactive at all times as compared with the males. During the day the moths are for the most part quiescent and become active just at dark. The male lives from 3 to 10 days and the female from 5—11 days.

Total life cycle :

The total life cycle of the pest varies from 28 to 34 days the egg, larval and pupal periods being 5 to 6, 18 to 22 and 5 to 6 days.:

Natural enemies :

The following parasites have been reported. (Cherian and Israel, 1939). An unidentified chalcid has also been reported in the egg stage of the pest. (2) *Megarhogas* n.sp. on fairly grown up caterpillars. (3) *Tricholyga sorbillans* Wied. a tachinid parasite on the body of the larva. (4) *Brachymeria euploeae* Westwr. a chalcid on the larva and *Goryphus nursei* Cam. an Ichneumonid parasite also on the larva.

Control measures :

Before the advent of synthetic chemicals the pest was controlled to a small extent by spraying lead arsenate or calcium arsenate at a strength of ½ oz. in one gallon of water or by dusting flowers of sulphur as a repellant. A small scale insecticidal trial was conducted in the orchard during 1956. Four treatments, namely, sprays of BHC 0·1%, DDT 0·1%, Aldrin 0·1% and Dieldrin 0·1% were tried on the infested plants and the population of caterpillars were counted before and after the teratment. From the trial it is evident that BHC 0·1% can effect a complete mortality of the caterpillars within 48 hours after treatment. Next in order of efficacy are Aldrin and Dieldrin respectively. The results of the trial are presented in Table I.

PUBLIC LIBRARY
1 APR 1962

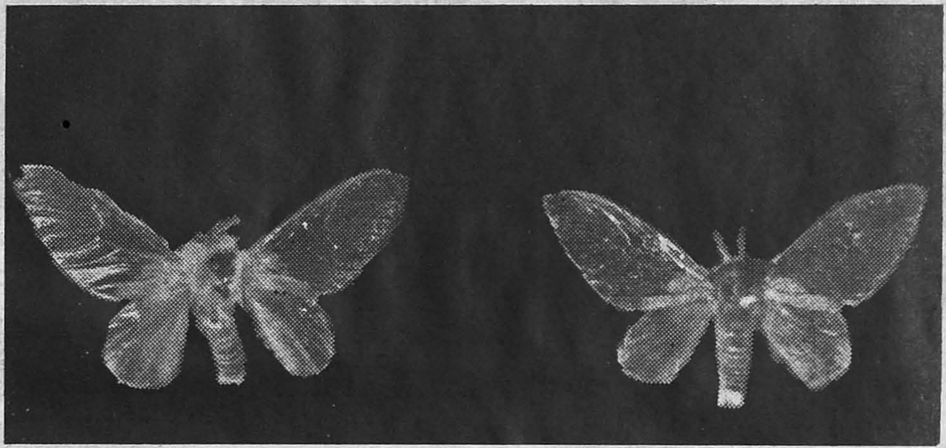
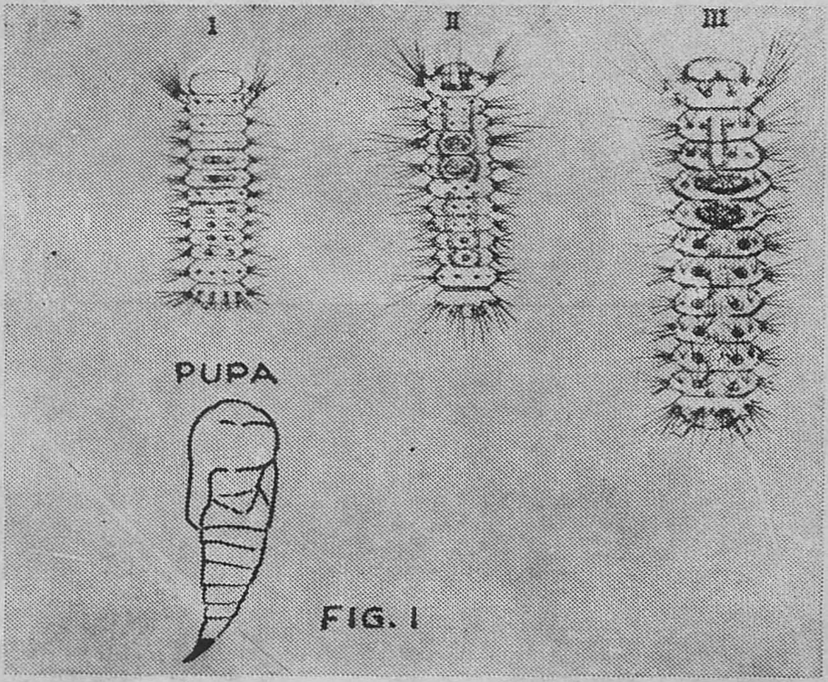


FIG. 1. Adult moths, *Perina nuda* Fabr.



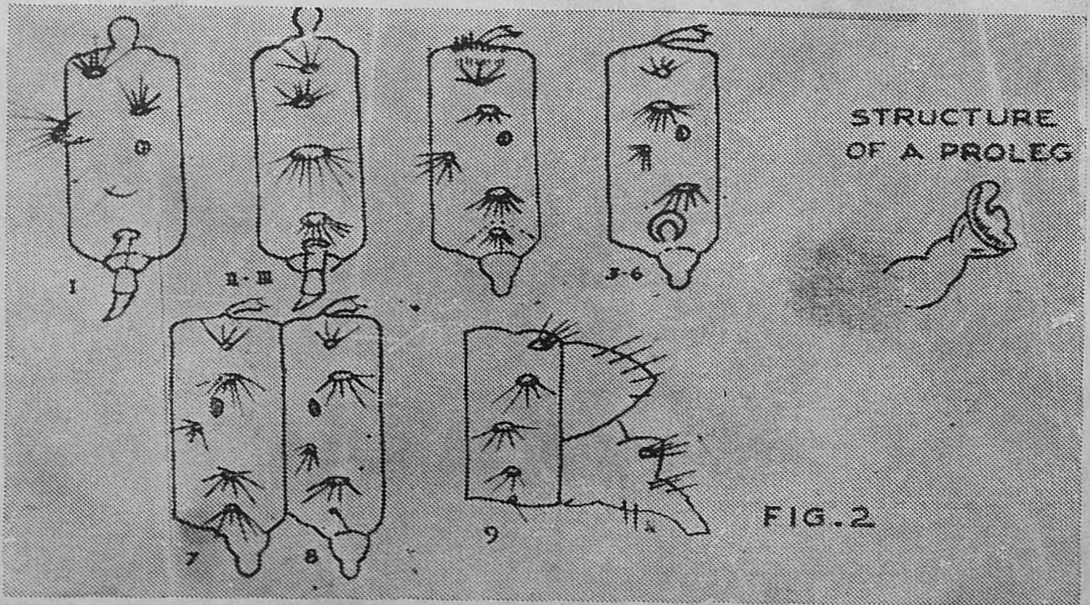


FIG. 1. Stages of the *Perina nuda* Fabr. larva and pupa, I to III represents young, medium sized and full grown larva. FIG. 2. Chaetotaxy and parts of thoracic and abdominal segments of *Perina nuda* Fabr. Larva I to III indicates the structure of thoracic legs and 3 to 9 the structure of abdominal legs.

TABLE I

Perina nuda, Fabr. Treatment and population counts

Treatments		Population before treatment	Population after treatment		Percentage of reduction in population
			24 hrs.	48 hrs.	
1. BHC 0·1%	..	12	4	0	100%
2. DDT 0·1%	..	14	9	7	55%
3. Aldrin 0·1%	..	10	6	4	60%
4. Dieldrin 0·1%	..	16	8	6	63%
5. Control	..	12	12	12	Nil

BIBLIOGRAPHY

1. Barlow, E. (1909) Miscellaneous Notes, *Indian Museum Notes*, 4: 14.

2. Cherian, M. C. and Israel, P. (1939) Notes on *Perina nuda*, Fabr (Lyman) and its natural enemies. *Madras agric*. J. 26: 203-207.

3. Fletcher, T. B. (1919) Annotated list of Indian Crop pests.

4. Hampson, G. F. (1892) *Proc. Third Ent. Meet.* Pusa 3: 93, *Fauna of British India Moths*, Thacker, Spink & Co., Ltd., (Bombay) 1: 486.

5. Lefroy, H. M. (1909) *Indian Insect Life*, Thacker, Spink & Co., 2, Creed Lane, London, 461.

6. Moore, F. (1883) *Lepidoptera of Ceylon*, L. Reeve & Co., 5, Henrietta *Street, Convent*, garden, 1882-3, 2: 93.

7. Ramachandra Rao, Y. (1951) Pests of fruit trees. *Proc. Second ent. Meet*, Pusa, 2: 251.

8. Ramakrishna Iyer, T. V. (1932) Annotated list of the insects affecting the important cultivated plants in South India. *Bull. Madras Agri. Dept.* 27:

J. Madras Univ., B, XXXI, No. 1, Pages 7-10, 1961.

Relations between Sums of Singular Series Associated with Certain Quadratic Forms

BY

K. ANANDA RAU

ABSTRACT

In this paper the equality is proved of the sums of singular series relating to quadratic forms whose coefficients satisfy certain conditions.

The singular series relating to the quadratic forms

$$a(x^2+y^2)+c(z^2+t^2)$$
$$a(x^2+y^2)+2c(z^2+t^2),$$

where x, y, z, t are integer variables and the coefficients a, c are odd positive integers, prime to each other, were considered in a recent paper [1], and certain expressions were obtained for the sums of the singular series. It is the purpose of this paper to show that we can use these expressions to prove the following theorem.

THEOREM: *Let a, c, a^*, c^* be odd positive integers such that $(a, c)=1$, $(a^*, c^*)=1$, $ac=a^*c^*$. Then the sums of the singular series relating to the forms*

$$a(x^2+y^2)+c(z^2+t^2) \qquad .. \quad (1)$$
$$a^*(x^2+y^2)+c^*(z^2+t^2) \qquad .. \quad (2)$$

are equal; and the sums of the singular series relating to the forms

$$a(x^2+y^2)+2c(z^2+t^2) \qquad .. \quad (3)$$
$$a^*(x^2+y^2)+2c^*(z^2+t^2) \qquad .. \quad (4)$$

are equal.

PROOF: Denoting the sums of the singular series relating to the forms (1), (2), (3), (4) respectively by

$$\frac{\pi^2 n}{ac}S(n), \quad \frac{\pi^2 n}{a^*c^*}S^*(n), \quad \frac{\pi^2 n}{2ac}S'(n), \quad \frac{\pi^2 n}{2a^*c^*}S^{*\prime}(n),$$

it is plain that in view of the hypothesis

$$ac=a^*c^*,$$

we have only to prove that

$$S(n) = S^*(n)$$
$$S'(n) = S^{*\prime}(n)$$

It will be convenient to recall some of the notations and definitions used in [1]. As in [1] we represent by k a variable running through all positive integers 1, 2, 3, ...; and we write

$$(a, k) = f, \ (c, k) = g, \ (2c, k) = w$$
$$a/f = \alpha, \ c/g = \gamma, \ 2c/w = \eta$$
$$\Phi_1(k) = -fgi^{f+g}, \ \Phi_2(k) = 4fgi^{\alpha+\gamma}$$
$$\Psi_1(k) = -fwi^{f+w}, \ \Psi_2(k) = 4fwi^{\alpha+\eta}$$
$$M = 2ac, \ N = 4ac, \ P = 8\,ac.$$

Analogously we define f^*, g^*, etc., by

$$(a^*, k) = f^*, \ (c^*, k) = g^*, \ (2c^*, k) = w^*,$$
$$a^*/f^* = \alpha^*, \ c^*/g^* = \gamma^*, \ 2c/w^* = \eta^*$$
$$\Phi_1^*(k) = -f^*g^*i^{f^*+g^*},$$
$$\Phi_2^*(k) = 4f^*g^*i^{\alpha^*+\gamma^*}$$
$$\Psi_1^*(k) = -f^*w^*i^{f^*+w^*},$$
$$\Psi_2^*(k) = 4f^*w^*i^{\alpha^*+\eta^*}$$
$$M^* = 2a^*c^*, \ N^* = 4a^*c^*, \ P^* = 8a^*c^*.$$

Now from $(a, c) = 1$, $(a^*, c^*) = 1$ it follows that for all values of k

$$fg = (k, a)(k, c) = (k, ac)$$
$$f^*g^* = (k, a^*)(k, c^*) = (k, a^*c^*),$$

and therefore since $ac = a^*c^*$ we have

$$fg = f^*g^*. \qquad .. \quad (5)$$

Similar reasoning shows that the hypotheses

$$(a, 2c) = 1, \ (a^*, 2c^*) = 1, \ ac = a^*c^*$$

lead to

$$fw = f^*w^*. \qquad .. \quad (6)$$

Further, applying (5) and the definitions of α, γ, α^*, γ^* we see that

$$\alpha\gamma = \frac{ac}{fg} = \frac{a^*c^*}{f^*g^*} = \alpha^*\gamma^*; \qquad .. \quad (7)$$

and applying (6) and the definition of α, η, α^*, η^* we get

$$\alpha\eta = \frac{2ac}{fw} = \frac{2a^*c^*}{f^*w^*} = \alpha^*\eta^*. \qquad .. \quad (8)$$

We saw in [1] that f, g, α, γ are odd for all values of k and that when k is odd we have w odd, and that when $k \equiv 0 \pmod{8}$ we have η odd. Similar reasoning shows that f^*, g^*, α^*, γ^* are odd for all values of k, and that when k is odd we have w^* odd, and that when $k \equiv 0 \pmod{8}$ we have η^* odd. It follows from these that for all values of k we have

$$(f-1)(g-1) \equiv 0, \qquad (f^*-1)(g^*-1) \equiv 0,$$
$$(\alpha-1)(\gamma-1) \equiv 0, \qquad (\alpha^*-1)(\gamma^*-1) \equiv 0,$$

the congruences being to modulus 4; and further when k is odd we have

$$(f-1)(w-1) \equiv 0, \qquad (f^*-1)(w^*-1) \equiv 0,$$

and when $k \equiv 0 \pmod{8}$ we have

$$(\alpha-1)(\eta-1) \equiv 0, \qquad (\alpha^*-1)(\eta^*-1) \equiv 0,$$

all the congruences being again to modulus 4. Using the above congruences and the equalities (5), (6), (7), (8) appropriately we obtain without difficulty

$$i^{f+g} = i^{fg+1} = i^{f^*g^*+1} = i^{f^*+g^*} \text{ for all } k \qquad .. \quad (9)$$

$$i^{\alpha+\gamma} = i^{\alpha\gamma+1} = i^{\alpha^*\gamma^*+1} = i^{\alpha^*+\gamma^*} \text{ for all } k \qquad .. \quad (10)$$

$$i^{f+w} = i^{fw+1} = i^{f^*w^*+1} = i^{f^*+w^*} \text{ for } k \text{ odd} \qquad .. \quad (11)$$

$$i^{\alpha+\eta} = i^{\alpha\eta+1} = i^{\alpha^*\eta^*+1} = i^{\alpha^*+\eta^*} \text{ for } k \equiv 0 \pmod{8} \qquad .. \quad (12)$$

Viewing the definition of $\Phi_1^*(k)$ given above together with (5), (9), and the definition of $\Phi_1(k)$ we see easily that for all values of k, and in particular when k is odd we have

$$\Phi_1^*(k) = \Phi_1(k). \qquad .. \quad (13)$$

Similarly, by appropriate use of (6), (7), (8), (10), (11), (12) we are able to prove that

$$\left.\begin{aligned} &\Phi_2^*(k) = \Phi_2(k) \text{ when } k \equiv 0 \pmod{4} \\ &\Psi_1^*(k) = \Psi_1(k) \text{ when } k \text{ is odd} \\ &\Psi_2^*(k) = \Psi_2(k) \text{ when } k \equiv 0 \pmod{8.} \end{aligned}\right\} \qquad .. \quad (14)$$

Since

$$M^* = M = 2ac, \quad N^* = N = 4ac, \quad P^* = P = 8ac$$

we get easily the analogues of the relations (20), (58) of [1], namely,

$$\left.\begin{aligned} S^*(n) &= \sum_{\varrho} \Phi_1^*(\varrho) R_M(\varrho) + \sum_{\xi} \Phi_2^*(\xi) R_N(\xi) \\ S^{*\prime}(n) &= \sum_{\varrho} \Psi_1^*(\varrho) R_M(\varrho) + \sum_{\tau} \Psi_2^*(\tau) R_P(\tau), \end{aligned}\right\} \quad .. \quad (15)$$

where, ϱ, ξ, τ run through the numbers prescribed in (21), (59) of [1]. By appropriate combination of (13), (14), (15) with (20), (58) of [1] we obtain easily.

$$S^*(n) = S(n)$$
$$S^{*\prime}(n) = S'(n),$$

as required to be proved.

REFERENCE

1. K. Ananda Rau: On the Summation of Singular series associated with certain Quadratic Forms (I). *J. Indian math. Soc.* 23: (1959) p. 65-96.

J. Madras Univ., B. XXXI, No. 1, Pages 11-35, 1961.

Studies on Indian Cryptonemiales—I

Grateloupia C.A.Ag.

BY

M. S. Balakrishnan

*University Botany Laboratory, Madras-5, India**

(Received for publication on 21-1-1961)

ABSTRACT

Details of reproduction are given for three species of *Grateloupia*, *G. comorinii*, *G. indica*, and *G. lithophila*. The study has shown a common pattern of post-fertilization development existing in all the three species.

Introduction

Schmitz (1883) stressed the importance of characters of cystocarp development in Floridean systematics. Since then the classification of the Florideophycidae has rested essentially on these characters. Kylin and Svedelius and others have emphasized the importance of a detailed study of the ontogeny of the cystocarp as a guide to the interpretation of the phylogeny of the group as a whole. There is, therefore, now a great need for intensive studies of the tropical red algae which have not received much attention so far.

The present accepted classification of the Florideophycidae is based mainly on that of Kylin (1932). In this system, the Cryptonemiales are separated from the Gigartinales by the fact that in the Cryptonemiales the auxiliary cell and the supporting cell of the carpogonial branch are formed on special accessory branches, while in the Gigartinales the auxiliary cells and the supporting cells of the carpogonial branches are modified intercalary cells of the normal vegetative branches themselves. Some workers have felt that this distinction is somewhat unsatisfactory (Fritsch, 1945, p. 656; Papenfuss, 1951, p. 10). In the light of

* Present Address: Department of Botany, Poona University, Poona-7, India.

present insufficient knowledge, especially of the developmental morphology of the different families of these two orders, it is rather difficult to assess the value of this character. Intensive studies on the comparative morphology of several members belonging to a single family and several families blonging to the same order are essential before any pronouncement can be made regarding the autonomy of these two orders (see Norris, 1957; also Drew, 1957). Among the Cryptonemiales the family Cryptonemiaceae shows a good deal of resemblance in vegetative characters to the families Nemastomaceae, Sebdeniaceae and Furcellariaceae of the Gigartinales (see Kylin, 1956, p. 216). The writer, therefore, selected this family (Cryptonemiaceae) for a detailed investigation as he felt that such an investigation would lead to a better understanding of the inter-relationships of the genera and families of the Cryptonemiales and possibly also of the Cryptonemiales with the Gigartinales.

There have been comparatively few detailed investigations of the developmental morphology of species belonging to the family Cryptonemiaceae till very recently. While the family is a large one with about twenty genera, detailed information regarding reproduction is available only for a few genera. Berthold (1884) was the first to devote attention to details of reproduction. In his monograph of the Cryptonemiaceae of the Gulf of Naples, he has given an illustrated account of the pre- and post-fertilization development in two species of *Grateloupia* and one of *Halymenia*. Schmitz and Hauptfleisch (1897) have in the 'Natürlichen Pflanzenfamilien', also given a short account of *Grateloupia filicina* (Lamour,) *C.Ag.* (=*Grateloupia filicina* (Wulf.) *C.Ag.*). Daines (1913), in his account of *Prionitis* attempted a detailed analysis of its reproductive structures. But, Sjöstedt's reinvestigation of the genus in 1926 showed that Daines' observations were incorrect in many respects. Sjöstedt (1926) also studied the developmental morphology of *Cryptonemia borealis* Kylin. In 1930 Kylin published a detailed and complete account of *Grateloupia fiilicina*, Subsequently Balakrishnan (1949) described the details of reproduction in an Indian species of *Grateloupia, G. lithophila* Boergs. Dawson (1954) in his "Marine Red Algae of Pacific Mexico" has given some details regarding reproductive structures and figures of some species of *Grateloupia* and *Halymenia* as well as of *Cryptonemia angustata* (Setchell *et* Gardner) Dawson, *Polyopes bushiae* Farlow (=*Carpopeltis bushiae* (Farlow) Kylin.), *Zanardinula andersoniana* (Eaton) Papenfuss (=*Prionitis*

andersoniana Eaton *ex* J. Agardh) and Z. *acroidalea* (Setchell *et* Gardner) Dawson.

Okamura (1907-13) in his 'Icones of Japanese Algae' has given a few figures of reproductive structures in Japanese species of *Grateloupia, Halymenia, Corpopeltis, Prionitis* and *Cryptonemia*, but he has not attempted an analysis of these structures. Recently, Kawabata has published a series of papers on the developmental morphology of Japanese algae belonging to this family: *viz.*, *Grateloupia turuturu* Yamada (Kawabata, 1954a); *Pachymeniopsis lanceolata* Yamada (Kawabata, 1954b); *P. yendoi* Yamada (Kawabata, 1957); *Cyrtymenia sparsa* Okamura (=*Phyllymenia sparsa* (Okamura) Kylin) (Kawabata, 1956) and an unidentified alga belonging to the Cryptonemiaceae (Grateloupiaccae) (Kawabata, 1955).

The family Cryptonemiaceae is well represented on the Indian coast. Four species of *Cryptonemia,* six of *Grateloupia,* ten of *Halymenia* and one each of *Polyopes, Carpopeltis* and *Corynomorpha* have been reported from the Indian region. At the kind suggestion of Prof. Iyengar, a comparative morphological study of this family was undertaken. Eight species have been studied in detail *viz., Grateloupia comorinii* Boergs., *G. indica* Boergs., *G. lithophila* Boergs., *Halymenia porphyroides* Boergs., *H. floresia* (Clem.) C.Ag., *H. dilatata* Zanard., *H. polydactyla* Boergs., and *Corynomorpha prismatica* (J. Ag.) J. Ag. The results of this investigation are presented in three parts. The present one deals with the genus *Grateloupia.*

Material and Methods

Material used in this investigation was provided by the writer's own collections as well as collections made at various times by Prof. M. O. P. Iyengar, Dr. T. V. Desikachary and Mr. V. S. Sundaralingam, all of whom were generous enough to place their collections at the writer's disposal. For the most part the material was preserved in 4% Formalin in sea water from which it was transferred by stages to 4% Formalin in fresh water before handling. Material preserved in Form-acetic-alcohol was also used, especially for microtoming. Most of the observations were made with squash preparations. Material was cut up into small pieces a few millimeters square and washed thoroughly in several changes of distilled water for 5-8 hours. Then the material was stained in a 1% aqueous solution of Erythrosin or Cyanosin for about five minutes; excess stain was drained off, a drop of 10%

glycerol in 4% Formalin added, and a coverslip placed on the material. After this, the material was slowly squashed with slight pressure exerted by the fleshy part of the thumb or the handle of a dissecting needle. This method yielded satisfactory preparations which, if sealed by ringing with Canada Balsam, lasted for a good length of time. Pretreatment with 10% potassium or sodium hydroxide or acids in various strengths as recommended by other workers, did not offer any particular advantage in the species studied. On the other hand, Norris' (1957) suggestion of using distilled water alone for 24 hours or more was found to be helpful in some cases, especially when the material had been killed and preserved in form-acetic-alcohol. Chopping into fine bits with a safety-razor-blade was also found useful in the case of recalcitrant materials.

Microtome preparations were also made, and these were found useful especially in confirming details studied by squash preparations. Microtome preparations were made by the paraffin method in the customary manner. Sections 10 μ thick were found most satisfactory and of all the stains tried, Haidenhain's iron-alum-haematoxylin gave best results. Observations made from microtome preparations were very useful in supplementing those made with squash preparations.

GRATELOUPIA C. A. AGARDH

Berthold (1884) has given a good account of *Grateloupia cosentinii* Kütz, (= *G. proteus* (Kunth.) Kütz.) and *G. dichotoma* J. Agardh. This account also includes some figures illustrating the essential features of reproduction in these two species. Subsequenly, Kylin (1930) gave a complete and detailed account of *G. filicina* (Wulf.) C.Ag. Balakrishnan (1949) dealt with the reproductive stages in the Indian species, *G. lithophila* Boergs. Finally, Kawabata (1954a) has given a detailed account of the Japanese species, *G. turuturu* Yamada. Okamura (1908), in his 'Icones of Japanese Algae' has figured carpogonial branches and auxiliary cells in *G. lancifolia* (Harv.) Okam., but a detailed and correct analysis of the reproductive structures is not given by him. Dawson gives figures of *G. schizophyla* Kütz. and *G. hancockii* Dawson. He also makes some remarks about reproduction in some more species in his account of the marine red algae of Pacific Mexico (Dawson, 1954).

In the present investigation, three species of *Grateloupia* occurring in the Indian region have been studied, *viz. G. comorinii*

Boergs, *G. indica* Boergs, and *G. lithophila* Boergs. The results of these studies are presented below.

Grateloupia comorinii Boergesen

Material: Cape Comorin, 28th October, 1957.

G. comorinii was described by Boergesen in 1938 and is based on specimens collected by Prof. Iyengar in September 1929 from Cape Comorin. The material used in the present investigation was collected by the writer from the type locality, and agreed in all respects with the type specimens in Iyengar's herbarium of South Indian Algae now located in the University Botany Laboratory, Madras.

The alga grows in abundance on rocks exposed to the surf, south east of the shore temple at Cape Comorin. Plants are mostly solitary, reaching a height of up to 30 (35) cm., and yellowish purple to brownish in colour, often with green specks, especially in the basal portions. The laminate fronds are highly variable in width and shape, tough in consistency and very slippery to touch on account of the highly lubricous surface. Attachment to the substratum is by a small discoid holdfast from which the single laminate frond arises. The blade is stipitate, the very short stipe gradually expanding into the foliaceous lamina. Occasionally clumps of more than one frond are seen, and in these instances, the fronds may issue from the discoid holdfast or from the lower part of the stipe (Plate III, fig. 3). The base of the frond is cuneate; the shape of the blade is very variable (see Boergesen, 1936, p. 217). Sometimes it is linear obovate to oblong-lanceolate and gradually tapering upwards to a blunt tip; in some it is rather abruptly truncate probably due to injury. Sometimes the blade is divided more or less deeply into two to four segments (Plate III, fig. 3). The laminae vary from one and a half to five times as long as broad. In general, the simple entire fronds are narrow while the truncate and divided blades are broad. The margins are more or less sinuate and undulating. Proliferations are almost always present, but the amount of proliferation varies. Some specimens are almost destitute of proliferations while others have a large number of proliferations from the upper parts, leaving the basal portion comparatively bare. Proliferations are usually most numerous and well developed along the upper margins of the older truncate specimens. The proliferations are elongate-elliptic and often have rather broad bases, tapering from the middle to the somewhat sharp tip. They may vary in size from quite small ones

(Plate III, fig. 4) to well developed ones which are about 2-2·5 cm. long and 5-7 mm. broad (Plate III, fig. 1).

At the time of his original description, Boergesen had only tetrasporic material. The writer was able to collect tetrasporic as well as mature cystocarpic specimens in October 1957. The cystocarpic plants are quite similar to the tetrasporic in general appearance, but they are slightly smaller in size, and almost always truncate with a large number of proliferations (Plate III, fig. 2). The cystocarps are scattered on the surface of the thallus in the older portions, the tips alone being free, and are seen as small dark specks when the plant is held up against the light. The cystocarps are almost totally immersed in the thallus, only the ostiolar portions projecting very slightly as minute conical protuberances. As a result, the cystocarpic plants have a slightly rougher feel as compared with the tetrasporic ones, though both are equally slimy.

Structure of the thallus :

As in all other Cryptonemiaceae, the thallus is multiaxial in construction, growth being of the fountain type. In section, the fronds are up to 400 μ thick, and show the typical structure found in the genus (Text-fig. 1). The medulla is proportionately large, and somewhat loose in texture. The inner medullary region is composed of loosely interwoven longitudinal filaments 5-9 μ in diameter, interspersed with slender rhizoids 3-5 μ in diameter and a large number of stellate cells, 15-20 μ in diameter and with arms up to 60 μ long. The stellate cells may be in direct connection with one another or may be connected by long slender rhizoids. The outer medulla is somewhat more compact and consists of slightly irregular cells 15-20 μ in diameter, and grading into the inner cortical region. The cortex is many layered, and clearly divisible into an inner and an outer cortex. The inner cortex is somewhat loose and is composed of fairly large and irregular cells 10-20 μ long and 6-9 μ broad grading almost imperceptibly into the outer medulla on the inner side; the cells get progressively smaller and more rounded as we approach the periphery of the thallus where they gradually merge into the outer cortex. The outer cortex is compact and composed of tightly packed dichotomous filaments, the cells of the filaments being rounded oblong and measuring 5-8 μ by 3-4 μ. The terminal cells of the cortical filaments are somewhat narrow and with rounded tips, 6 μ long and 3 μ wide, densely pigmented and forming a sort of surface palisade

layer. Quite often some of these cells grow out into unicellular hairs 2-2·5 μ wide and up to 150 μ long.

Development of the carpogonial branches:

The carpogonial branches develop in special branch systems, the so-called 'ampullary clusters' or 'ampullae' (Sjöstedt, 1926). Each ampullary cluster arises as an accessory branch system from a normal vegetative cell in the inner cortex (Text-fig. 2); it is, therefore, secondary in origin. The primary branch of the cluster (the primary ampullary filament) is four- to six-celled and is incurved or slightly coiled. The cells of the primary ampullary filament are subspherical, measuring 6-9 μ diameter. From the cells of this primary ampullary filament five- to ten-celled erect secondary branches are given off towards the surface of the thallus; occasionally, from these secondary erect branches, branches of a higher order may also be produced. The cells of the erect secondary ampullary branches are somewhat oblate-spheroidal to inverted conical, 6-9 μ wide at the base of the branches and getting gradually smaller towards the tip. The terminal cells of the secondary ampullary branches are somewhat cylindrical or subconical, 7-9 μ long and 3-4 μ broad. The fully formed ampulla is a hollow, conical flask-shaped structure, 20-25 μ wide at the base and 10-15 μ wide at the top; the bottom of the flask is formed by the slightly coiled primary ampullary filament and the sides by the erect secondary branches, which reach almost to the surface of the thallus.

The carpogonial branch normally arises from an intercalary cell (Text-fig. 6 *sc*) of the primary ampullary filament. The carpogonial branch is two-celled, consisting of a terminal carpogonium (Text-fig. 6 *cp*) and a single hypogynous cell, the carpogonial-branch-cell (Text-fig. 6 *cb*). As the ampulla matures, the carpogonium develops a long trichogyne; a short three- to five-celled lateral sterile branchlet is developed from the carpogonial branch cell (Text-fig. 6 *st*). The trichogyne originates as a small papillate structure which ultimately develops into the thick walled long trichogyne. The trichogyne is 3 μ broad and projects appreciably above the surface of the thallus (Text-fig. 6). The length of the trichogyne is variable, but it is usually 12-15 times as long as the carpogonium. The carpogonium is short and conical with a curved base, measuring 6·5 μ wide and 7 μ high. The trichogyne is slightly constricted at the base. Because of the incurving or coiling of the primary ampullary filament, the carpo-

gonium usually occupies the centre of the ampullary cluster at the base.

Development of the auxiliary cells:

As in all Cryptonemiales, here also the auxiliary cells are distinguishable even before fertilization. Like the carpogonial branches, the auxiliary cells also occur in special ampullary clusters which arise as accessory laterals from cells of the inner cortex (Text-fig. 2). The general structure of the auxiliary cell ampullae is quite similar to that of the carpogonial ampullae (Text-fig. 3). They show a 5-8 celled primary filament which is slightly incurved and from which erect secondary branches arise, finally forming a hollow conical flask-shaped structure. The cells of the primary ampullary filament are spherical and 7·5-9 μ in diameter. The secondary branches are six to ten cells long; the basal cells of the secondary branches are flattened spherical, and 6 μ high and 9 μ in diameter; cells higher up get progressively smaller and are flattened spherical or inverted conical in shape and are 4-5 μ × 8-9 μ in size. The terminal cells of the secondary ampullary branches are cylindrical or conical and are 7-9 μ long and 3-5 μ wide. The auxiliary cell ampullae are slightly larger and more profusely branched than the carpogonial ampullae, and are 30-35 μ wide at the base and 20-25 μ wide at the narrowed top portion. The auxiliary cell is an intercalary cell of the primary ampullary filament (Text-fig. 3 *au*), or occasionally, the lower most cell of one of the erect secondary ampullary branches. It is much larger than the other cells in the ampullary cluster; it is spherical to ovoid, 10-12·5 μ in diameter or 10-12·5 μ × 8-11 μ, uninucleate and densely protoplasmic. The auxiliary cell occupies the approximate centre of the ampullary cluster at the base.

Post-fertilization development:

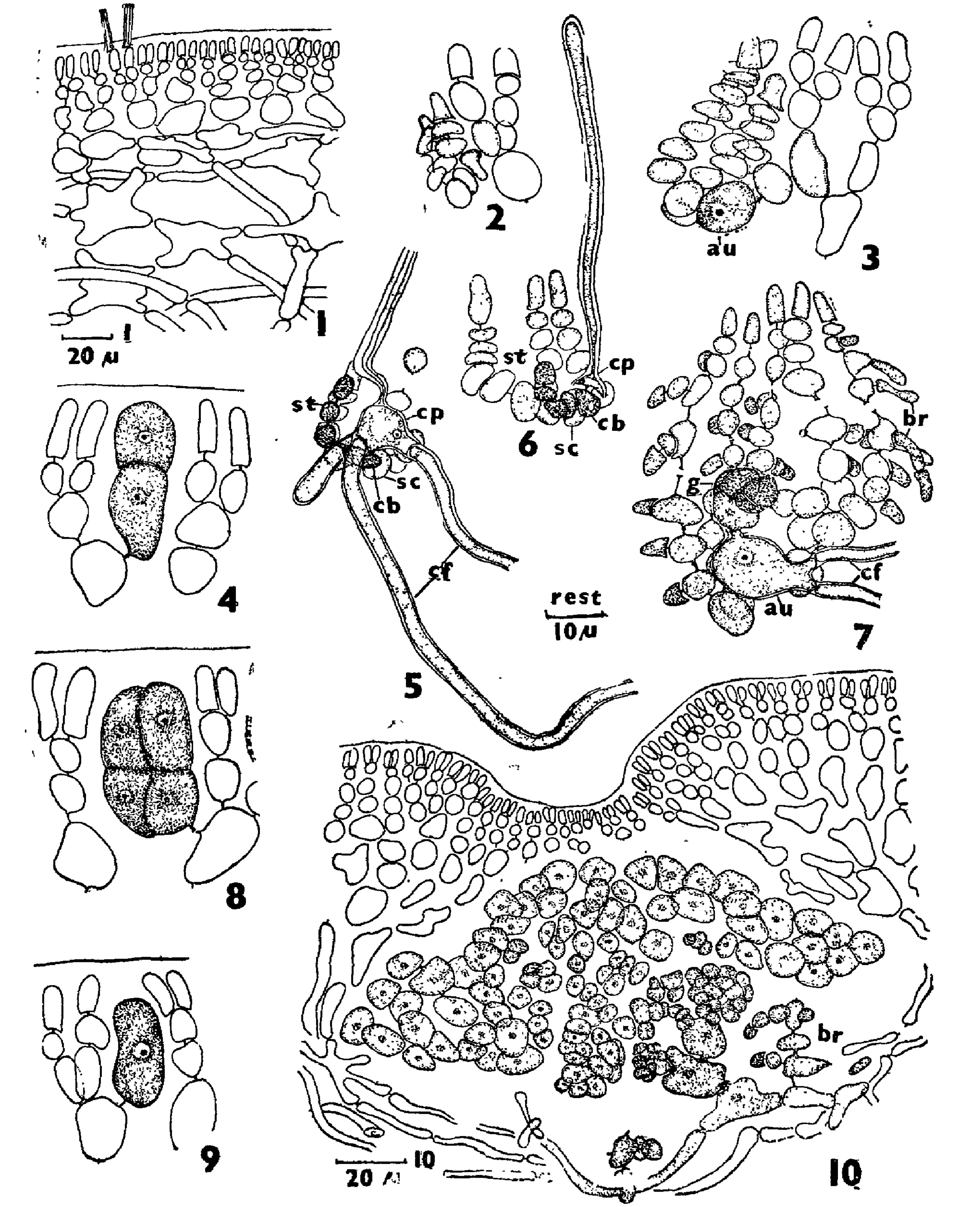
After fertilization, the trichogyne is first cut off from the rest of the carpogonium. The fertilized carpogonium now enlarges and produces a number of connecting filaments directly, without fusing with any cell or cells of the carpogonial ampulla (Text-fig. 5). This observation of the writer agrees with that of Kylin (1930) who found in *G. filicina* that the fertilized carpogonium enlarges and directly sends out connecting filaments without fusing with any cell of the carpogonial ampulla. In the present species the writer has been able to observe very clearly early stages in post-fertilization development. As is seen from Text figure 5, it is only the carpogonium which enlarges and sends out

connecting filaments without prior or later fusion with any cell of the carpogonial ampulla. In all the cases examined, the carpogonial branch cell and its lateral sterile branch could always be made out in their entirety (Text-fig. 5, *cb*, *st*) *G. comorinii* further resembles *G. filicina* in that the enlarged carpogonium still retains a fairly regular shape (*cf.* Kylin, 1930, fig. 10c, with Text-fig. 5 of the present paper).

At this stage of post-fertilization development, there is an impoverishment of the contents of the cells of the primary and secondary ampullary branches. There is also a slight widening of the protoplasmic (pit) connections between the cells in the vicinity of the fertilized carpogonium. It appears likely that the enlarging carpogonium draws nourishment from the sterile branches of the ampullary cluster in order to sustain its further development.

The connecting filaments lack cross walls; they run in all directions, seeking out the fully developed auxiliary ampullae and fusing with the auxiliary cells in them. Usually there is a slight swelling at the point of fusion. After fusing with an auxiliary cell the connecting filament frequently continues its growth, seeking out other auxiliary ampullae and fusing with the auxiliary cells in them. The auxiliary cell, after fusion with a connecting filament, and presumably after receiving a diploid nucleus from it, enlarges slightly and divided transversely by a concave wall. The upper lenticular cell thus formed is the gonimoblast initial. This by further divisions produces the gonimoblast. The developing gonimoblast is seen to consist of several distinct clusters or groups of gonimoblast filaments (gonimolobes) in various stages of development (see Kylin 1956, p. 215; Fritsch, 1945, p. 365).* The lower daughter cell resulting from the first division of the auxiliary cell enlarges and becomes irregular on account of fusion with adjacent cells of the primary ampullary filament; the pit connection between it and the gonimoblast initial is quite large and prominently seen. The lower cells of the gonimoblast filaments remain sterile; the upper cells become carposporangia, each carposporangium producing a single carpospore. The carpospores are somewhat polyhedral with rounded corners about 10-15 μ in

* The term 'Gonimolobe' is used here as was used by Schmitz (1883) and other earlier authors (Kylin, *l.c.*, Fritsch, *l.c.*) and not in the sense indicated by Drew, 1954 (see also Prakasa Rao, 1956).



TEXT-FIGS. 1-10: *Grateloupia comorinii* Boergs.

1. Longitudinal section of the thallus showing internal organization. 2. Young ampullary cluster, developing as an accessory lateral branch from a cortical filament. 3. Fully developed auxiliary ampulla; note that the auxiliary cell is an intercalary cell of the primary ampulary filament. 4. Tetrasporangium after the first division. 5. Fertilized carpogonium sending out three connect-

diameter. They are uninucleate and contain a number of ribbon-shaped chromatophores.

Side by side with gonimoblast development, the erect secondary ampullary branches develop lateral branches in some profusion. These newly produced branches are few to several cells long, and are more or less horizontal, with the tips directed downwards (Text-fig. 7,br). By interlacing, these branches soon form an intricate network surrounding the developing gonimoblast. During later stages of gonimoblast development, this 'basket' surrounding the gonimoblast is gradually destroyed, and finally only scattered remnants of it are seen in the mature cystocarps. Production of these branches is evidently for nutritive purposes as is seen from the fact that these get gradually impoverished and finally destroyed altogether during the gonimoblast development.

The ripe cystocarps are flattened subglobose, up to 200 μ in diameter, 100-125 μ high, and are completely sunk in the thallus; there is a central ostiole at the top through which the ripe carpospores finally escape (Text-fig. 10).

Development of tetrasporangia :

The tetrasporangia arise as secondarily developed one-celled lateral branches from the cells of the inner cortex. The tetrasporangium is cut off by a curved wall from the top portion of the mother cell. It soon enlarges and attains an elongate ellipsoidal shape (Text-fig. 9). After reaching a fairly large size, it divides transversely into two (Text-fig. 4). Each of the two daughter-protoplasts thus formed, divides vertically once again, so that four spores result (Text fig. 8). Depending on whether the vertical divisions are both in the same plane or perpendicular to each other, the tetraspores are cruciately or decussately arranged. Ripe tetrasporangia are ovoid to ellipsoidal in shape and 25-30 μ long

ing filaments; note that it still retains a regular shape; note also that the carpogonial branch cell (*cb*) and its lateral sterile branch (*st*) still retain their identity. 6. Mature carpogonial ampulla. 7. Auxiliary ampulla showing an early stage in gonimoblast development and the initiation of the lateral branchlets from the cells of the secondary ampullary branches. 8. Mature tetrasporangium. 9. Young, unidivided tetrasporangium. 10. Ripe cystocarp in section.

(*au*: auxiliary cell; *br*: nutritive side branchlets; *cb*: carpogonial branch cell; *cf*: connecting filament; *cp*: carpogonium; *g*: gonimoblast; *sc*: supporting cell; *st*: lateral sterile branch).

and 12-16 μ broad. They are superficially situated in the cortex and scattered all over the thallus surface, with the exception of the parts just behind the apices. The ripe tetraspores are uninucleate, densely protoplasmic and contain several ribbon-shaped chromatophores.

Grateloupia indica Boergesen

Material: Okha, 6th January, 1955, *leg.* T. V. Desikachary.

Grateloupia indica was first described by Boergesen in 1932 based on a specimen cast ashore at Okha port. The material used in the present investigation was collected from the type locality and comparison with Boergesen's specimens in Iyengar's herbarium at the University of Madras has confirmed its identity. Both tetrasporic and cystocarpic specimens were present in the collection.

The alga grows on rocks facing the open sea. It is a large alga, plants often reaching a length of one metre (see Boergesen, 1932), though smaller specimens are more common. The basal disc is rather small for a plant of its size, and the single erect frond arising from it is distinctly stipitate. The stipe is short, cuneate, and expands rather abruptly into the flat oblong — linear blade which is much divided. The blade is very tough in consistency, with a highly lubricous surface, making it very slippery to touch; the colour of the living plant is purple, but dried specimens appear dark purple with a violet tinge in the older portions, slightly paler in the upper parts. Division of the lamina is very irregular, resulting in lobes of various sizes and shapes (Plate IV, fig. 5). The overall width of the blade ranges from 25 cm. to 30 cm; the segments vary from 5 cm. to 10 cm. in width. In old cystocarpic plants the blades show a large number of perforations of varying sizes. These perforations probably originate from empty cystocarpic cavities. The margin of the blade is usually irregularly sinuate and undulate. Proliferations are not very common and are usually seen only in old blades which have been damaged or are highly perforated. When present, they are quite numerous, small and spathulate.

Structure of the thallus:

The blades are 300-350 μ thick, multiaxial in construction, and in section show the typical structure of *Grateloupia* (Text-fig. 11). The medulla is somewhat large and more or less loose in texture,

just as in *G. comorinii.* The inner medulla is composed of longitudinal thick-walled medullary filaments, 5-8 μ in diameter, and interspersed with stellate cells and slender rhizoids. The stellate cells are about 15-20 μ in diameter with arms up to 75 μ long; the rhizoids are 3-4 μ thick. The stellate cells may be in direct connection with one another or may be connected by rhizoids. The outer medulla is more compact than the inner and consists of large irregular cells measuring 15-30 μ × 10-12 μ, and gradually merged into the inner cortex. The cortex is many layered and clearly divisible into an inner and an outer cortex. The cells of the inner cortex are more loosely arranged than in the outer cortex. Cells of the innermost layer are large and somewhat irregular, 10-15 μ long and 10-12 μ broad. As we go towards the outer cortex, the cells get progressively smaller and more rounded. The outer cortex is composed of densely packed dichotomous cortical filaments with cells somewhat rounded and measuring 5-8 μ by 3-5 μ. The terminal cells of the cortical filaments are narrow and cylindrical and 6-9 μ long and 3-4 μ broad with somewhat rounded tips. They are densely pigmented and form a photosynthetic surface palisade layer.

Development of the carpogonial branches:

As in *Grateloupia comorinii,* here also the carpogonial branches are developed in special 'ampullae', which are accessory laterals of the inner cortical cells. The primary ampullary filament is four to six cells long, the cells being subspherical and 6-9 μ in diameter. From the cells of the primary ampullary filament, erect secondary ampullary branches arise. The cells of these secondary branches are rounded, or inverted conical, getting progressively smaller towards the tips; the basal cells of these branches are usually 5-6 μ in diameter and the subterminal cells 3-5 μ in diameter. The terminal cells are cylindrical or conical, 7-10 μ long and 3-3·5 μ broad. As in *G. comorinii,* the fully developed ampulla is a conical flask-shaped structure, broad at the base and narrowing towards the top, which reaches up to the level of the outermost cortical layer. The entire cluster is 20-25 μ wide at the base and 10-15 μ wide at the top.

The supporting cell of the carpogonial branch is an intercalary cell of the primary ampullary filament. The carpogonial branch is two-celled; the carpogonial branch-cell always has a short two- to five-celled lateral sterile branch (Text-fig. 18 *st*). The carpogonium is short, subconical, 5-8 μ wide at the base and 5-7·5 μ

high. The trichogyne is much longer than in *G. comorinii*, often up to 25 times as long as the carpogonium, and occasionally twisted or coiled; the trichogyne is 2·5-3·5 μ wide, and has a thick wall. The basal constriction is very pronounced. The primary ampullary filament is curved, so that the carpogonial branch occupies approximately the centre of the ampullary cluster, with the long trichogyne pushing through vertically and projecting appreciably above the surface of the thallus (Text-fig. 15).

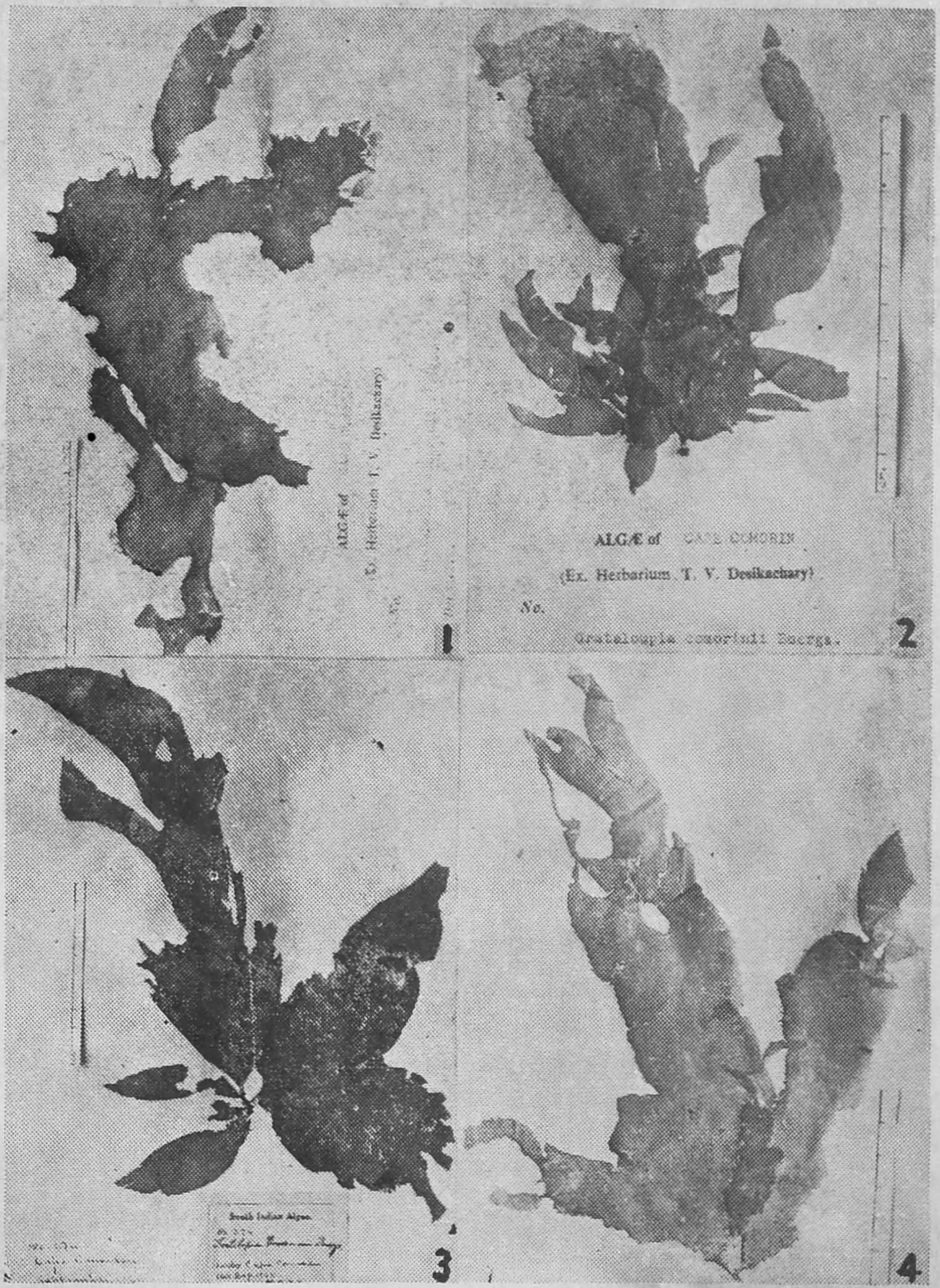
Development of the auxiliary cells:

The auxiliary ampullae also arise as accessory lateral branches of the inner cortical cells. Their general construction is quite similar to that of the carpogonial ampullae, but they are, as a rule, much more profusely branched than the carpogonial ampullae. The primary ampullary filament is five to eight cells long, the cells being subspherical and 8 μ in diameter. The erect secondary ampullary branches are up to ten cells long; the basal cells of the secondary branches are 6-8 μ in diameter and the subterminal cells 3-5 μ in diameter. The terminal cells are cylindrical or conical, 7-10 μ long and 3-3·5 μ wide. The fully developed auxiliary ampullae are 30-35 μ wide at the base and 20-25 μ wide at the top.

The auxiliary cell is an intercalary cell of the primary ampullary filament (Text-fig. 12) or the basal cell of one of the erect secondary ampullary branches (Text-fig. 13). It is ovoid to subspherical, 10-13μ long and 7·5-10μ wide, densely protoplasmic and uninucleate. The primary ampullary filament is curled so that the auxiliary cell occupies the approximate centre of the ampulla in the basal part.

Post-fertilization development:

After fertilization, the trichogyne is first cut off. The fertilized carpogonium then enlarges and ultimately becomes a much swollen, irregularly lobed structure; from the tips of the lobes, connecting filaments are produced (Text-fig. 15). Here also, it could clearly be observed that it was only the carpogonium which had enlarged considerably and developed into an irregular lobed cell; there was no fusion of the carpogonium with any adjacent cell or cells of the ampulla either prior to or after fertilization. As Text-figure 15 shows, the carpogonial branch cell and its short lateral sterile branch preserve their identity and can be clearly made out (Text-fig. 15, *cb, st*) at this stage. As compared with *G.*



FIGS. 1–4 *Grateloupia comorinii* Boergs.

1. Topotype tetrasporic specimen collected in October 1957 showing numerous minute proliferations all along the margin. 2. Topotype cystocarpic specimen collected from Cape Comorin in October 1957, showing a large number of proliferations. 3. Isotype tetrasporic specimen (No. 274) collected from Cape Comorin by Iyengar in September 1924, showing several blades arising from a common stipe. 4. Topotype cystocarpic specimen collected in October 1957 showing irregular margin and truncation due to injury.

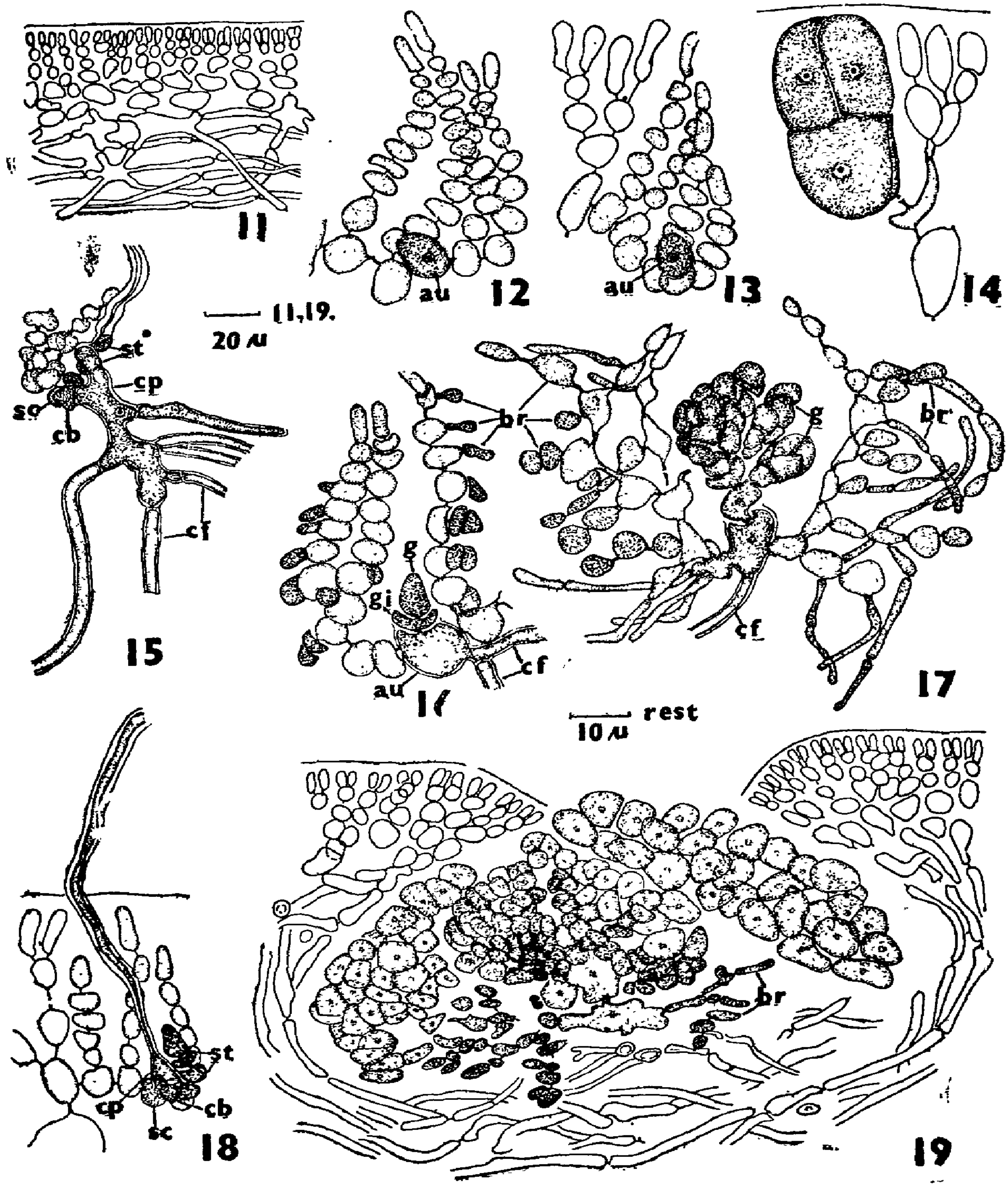
CONNEMARA PUBLIC LIBRARY
9 - AUG 1965

FIGS. 5 AND 6.

FIG. 5. *Grateloupia indica* Boergs. Topotype tetrasporic specimen collected from Okha by Desikachary in January 1955. FIG. 6. *Grateloupia lithophila* Boergs. Isotype tetrasporic specimen (No. 370) collected by Iyengar from Royapuram, Madras, in January 1926.

CONNEMARA PUBLIC LIBRARY
9 - AUG 1965

comorinii, a greater widening of the protoplasmic connections between the fertilised carpogonium and the connected cells of the system is seen in this species. There is also a greater impoverishment



TEXT-FIGS. 11-19: *Grateloupia indica* Boergs.

11. Longitudinal section of the thallus showing the internal organization. 12. Auxiliary ampulla; note that the auxiliary cell (*au*) is an intercalary cell of the primary ampullary filament. 13. Auxiliary ampulla; note that the auxiliary cell (*au*) here is the basal cell of a secondary ampullary branch.

of the contents of the cells of the ampullary branches. This is perhaps due to the fact that in *G. indica* the fertilized carpogonium enlarges much more than in *G. comorinii*, becoming greatly lobed and sending out a much larger number of connecting filaments (cf. Text-fig. 15 with Text-fig. 5); this would, obviously, mean a greater drain on the cells which remain in organic connection with it. The connecting filaments are non-septate, thick-walled and 2-3 μ in diameter. They run in all directions, seeking out auxiliary cells and fusing with them. After fusing with one auxiliary cell, the connecting filament in many cases continues its growth, seeking out successive auxiliary ampullae and fusing with the auxiliary cells in them.

After fusion with the connecting filament, and presumably after receiving a diploid nucleus from it, the auxiliary cell cuts off the gonimoblast initial at the top by a curved wall. By further division, the gonimoblast initial produces the gonimoblast, which is densely clustered and divided into a number of gonimolobes. As in *G. comorinii*, here also the lower derivative of the auxiliary cell gradually fuses with the neighbouring cells of the primary ampullary filament and an irregular fusion cell results (Text-fig. 17). This irregular fusion cell is prominently seen at the base of the gonimoblast in the ripe cystocarps (Text-fig. 19).

A network of tertiary and quaternary branches is developed from the ampulla during the early stages of gonimoblast development just as in *G. comorinii*. This nutritive 'basket' fully envelopes the gonimoblast. Early stages in the production of this network of nutritive branches could be easily followed in this species. The initiation of these branches appears to coincide with fusion of the connecting filament with the auxiliary cell, or in some cases, with the cutting off of the gonimoblast initial (Text-fig. 16). The tertiary branches are up to six cells long, slender

14. Mature tetrasporangium. 15. Fertilized carpogonium sending out six connecting filaments; note its large size and lobed nature; note also that the carpogonial branch cell (*cb*) ad its lateral sterile branch (*st*) retain their identity. 16. Auxiliary ampulla showing an early stage in gonimoblast development and the initiation of the lateral nutritive branchlets (*br*). 17. A later stage showing the gonimoblast clearly divided into two gonimolobes and the well developed lateral nutritive branchlets. 18. Carpogonial ampulla. 19. Ripe cystocarp in section.

(*au*: auxiliary cell; *br*: nutritive branchlets; *cb*: carpogonial branch cell; *cf*: connecting filament; *cp*: carpogonium; *g*: gonimoblast; *gi*: gonimoblast initial; *sc*: supporting cell; *st*: lateral sterile branch).

and horizontal, slightly curving downwards (Text-fig. 17); the cells of these branches are long and narrow; the quaternary branchlets arise almost at right angles from these slender tertiary branches, and the cells of these branchlets are quite short and rounded (Text-fig. 17). Only remnants of these are seen in the mature cystocarp, as they get used up during the later stages of gonimoblast development.

The ripe cystocarps are subspherical to spherical, occasionally somewhat flattened, about 250 μ in diameter, and 150 to 225 μ high. The cystocarps are scattered, and totally sunk in the thallus. There is a central ostiole through which the ripe carpospores finally escape (Text-fig. 19). Ripe carpospores are polyhedral with somewhat rounded corners, 18-20 μ × 10-15 μ, uninucleate and contain several short ribbon-shaped chromatophores.

Development of tetrasporangia :

Just like the carpogonial and auxiliary ampullae, the tetrasporangia are secondarily developed laterals of the cells in the inner cortex, but they are single-celled. The tetrasporangium initial is cut off by a curved wall from the top portion of the mother cell. It enlarges considerably and becomes cylindrical or ellipsoidal in shape. The first division of the tetrasporangium is transverse, resulting in two uninucleate protoplasts one above the other. Each of these daughter protoplasts then divides vertically and four tetraspores are formed (Text-fig. 14). The tetraspores are cruciate or decussate depending on whether the planes of the second division coincide or are at right angles to each other. The ripe tetrasporangia are much larger than in *G. comorinii*. They are sub-ovoid in shape, 35-40 μ long and 15-20 μ wide. The tetraspores are uninucleate, densely protoplasmic and contain a number of parietal ribbon-shaped chromatophores.

Grateloupia lithophila Boergesen

Material: Madras harbour, 20th October 1940, 10th July 1941.

Grateloupia lithophila was described by Boergesen in 1938 and is based on specimens collected from Madras (Madras Harbour) by Prof. M.O.P. Iyengar in 1925. Boergesen's type specimen was tetrasporic. Sexual plants were not seen by him. Subsequently, Prof. Iyengar and the writer have collected sexual material. The alga is dioecious. The writer described in 1949 some aspects of reproduction in this species but a few stages could not be obtained at the time. In the present account spermatangia and some more details in the development of the cystocarp are

described. The material used in this investigation was collected from the type locality.

The alga grows immediately north of the Madras Harbour (the type locality) and also at Mahabalipuram (Srinivasan, 1946), on stones exposed to strong surf action. The plants are markedly caespitose, growing in thick short clumps up to 15 cm. high, and firmly attached to the substratum. The basal discoid holdfast is small and the several erect blades which arise from it in a clump are subsessile, simple or divided and linear-lanceolate, tapering from the middle to the extremities (Plate IV, fig. 6); they are either flat or markedly sinuate or undulating. The breadth of the blades varies from 0·25—1 cm. and the margins are usually devoid of proliferations. However, specimens in which the upper end has become truncate and rounded owing to injury, show a number of proliferations usually issuing from the truncate tip. These proliferations are up to five cm. in length and resemble the main fronds in shape. The plants are generally olivaceous to violet; the surface is highly lubricous and very slippery to the touch.

Structure of the thallus:

The thallus is multiaxial in construction, 200—250 μ thick, and has the typical internal organization of *Grateloupia* (Text-fig. 20). The inner medulla is composed of loosely interwoven thick-walled longitudinal (medullary) filaments, 4-7 μ wide, intermingled with rhizoids and stellate cells. The stellate cells are 12-15 μ in diameter, with arms 40-50 μ long; they are either in direct connection with one another or are connected by slender rhizoids which are 2·5—3 μ in diameter. The outer medulla is composed of somewhat loosely arranged irregular cells 15-20 μ long and 12—15 μ broad. The cortex is six- to eight-layered. The cells of the inner three layers are somewhat large, 8-12 μ long and 4-8 μ broad, grading into the outer medulla on one side and the outer cortex on the other. The outer cortex is composed of compactly packed dichotomous cortical filaments, the cells of which are somewhat rounded, 5-6 μ long and 3-4 μ broad. The terminal cells of the cortical filaments are subcylindrical with rounded tips 4-6 μ long and 2·5-3 μ broad and posses a number of small ribbon-shaped chromatophores. These cells form the surface photosynthetic layer.

Development of Spermatangia:

The alga is dioecious. The male plants are smaller than the cystocarpic or tetrasporic ones, reaching only about 10 cm. in

height; the blades also are somewhat narrower, the maximum width being usually about 1 cm. The spermatangial sori occur as irregular patches in the younger portions of the thallus just behind the tips, and are not easily distinguished in living specimens. In these sori, the outermost cortical cells are somewhat elongated and function as spermatangial mother cells. The spermatangia are terminal on the elongated mother cells; usually only one is seen on each mother cell (Text-fig. 23) very rarely two. The ripe spermatangia are ovoid spherical, 2·5-4 μ in diameter and show the typical organisation of the Floridean spermatangium. The nucleus occupies the upper end and is situated above a large posterior vacuole; the protoplasm forms a thin lining to the wall of the spermatangium. The spermatia escape by squeezing out through the gelatinized walls above them. Their passage is clearly discernible (Text-fig. 23) in the clear, gelatinized empty patches in the sori.

In the aggregation of spermatangia into regular sori *G. lithophila* resembles *G. dichotoma* (Berthold, 1884, Pl. 7, fig. 8) and *G. howei* Setchell *et* Gardner (Dawson, 1954, Pl. 1, fig. 2). In the two former species, the fertile cortex is unmodified, while in *G. howei* the cortex in the fertile region is nemathecially modified.

Development of the carpogonial branches:

As in the other two species earlier described here, the carpogonial branches in *G. lithophila* are formed in ampullae which arise as accessory laterals of the inner cortical cells. The primary ampullary filament is four-to six-celled; the cells are subspherical and 6-8 μ in diameter. Erect secondary ampullary branches arise from cells of the primary ampullary filament. The basal cells of these branches are subspherical, 5 μ in diameter; the upper cells get progressively smaller towards the tip, the subterminal cells being 3·5-4 μ in diameter; these upper cells of the secondary ampullary branches are subspherical, flattened spherical, or occasionally inverted conical. The terminal cells of the secondary ampullary branches are elongate cylindrical with rounded tips, or subconical, up to 10 μ long and 4 μ wide. The fully developed ampullae are 20-25 μ wide at the base and 10-15 μ wide at the top. The supporting cell of the carpogonial branch is an intercalary cell of the primary ampullary filament (Text-fig. 22, *sc*). The carpogonial branch is two celled as in the other previously described species; the carpogonial-branch-cell always has a short, 3-6 celled lateral sterile branch (Text-fig. 22, *st*). The trichogyne

develops from the carpogonium as a small papillate structure (Text-fig. 22). Later on it increases very much in length, and when fully developed, is eight to fifteen times as long as the carpogonium, projecting appreciably above the surface of the thallus (see Text-figs. 1-3, and Plate I, fig. 1 in Balakrishnan, 1949). The carpogonium is conical, 8-9 μ wide at the base, and 5-7·5 μ high. The trichogyne is 3·5-4 μ wide, and thick-walled; in the basal portion, the thickening of the wall is so great that the contents are reduced to a slender protoplasmic strand (see Text-figs. 3, 4 in Balakrishnan, 1949).

Development of the auxiliary cells:

The auxiliary ampullae are also accessory laterals of the cells of the inner cortex. These are more profusely branched but otherwise resemble the carpogonial ampullae in structure. The primary ampullary filament is six to eight cells long; the cells are subspherical and 7-9 μ in diameter. From the cells of the primary ampullary filament erect secondary ampullary branches arise; these are up to ten cells long. The basal cells of the secondary ampullary branches are subspherical, 5-6·5 μ in diameter; cells higher up are progressively smaller, the subterminal cells being 4-4·5 μ in diameter. The upper cells of these branches are either subspherical, flattened spherical or inverted conical. The terminal cells of the secondary ampullary branches are elongate-cylindrical or conical and 10 μ long and 4-5 μ wide. The auxiliary cell is an intercalary cell of the primary ampullary filament or the basal cell of one of the secondary ampullary branches. It is slightly larger than the other cells in the ampulla and is ovoid to subspherical and 8-10 μ in diameter, and densely protoplasmic and uninucleate (Text-fig. 21) (see also Text-figs. 6, 9 in Balakrishnan, 1949). The mature auxiliary ampulla is more or less conical and is 25-30 μ wide at the base and slightly narrower at the top.

Post-fertilization development:

After fertilization, the trichogyne is first cut off. The fertilized carpogonium then enlarges to form a large, irregular and densely protoplasmic cell from which two to several connecting filaments are given out (Text-fig. 26). In 1949, the writer had stated that he was unable to determine whether this large cell is formed by the enlargement of the carpogonium alone or whether it is due to the fusion of the fertilized carpogonium with one or more cells of the carpogonial ampulla. In the present investigation, several stages in early post fertilization development

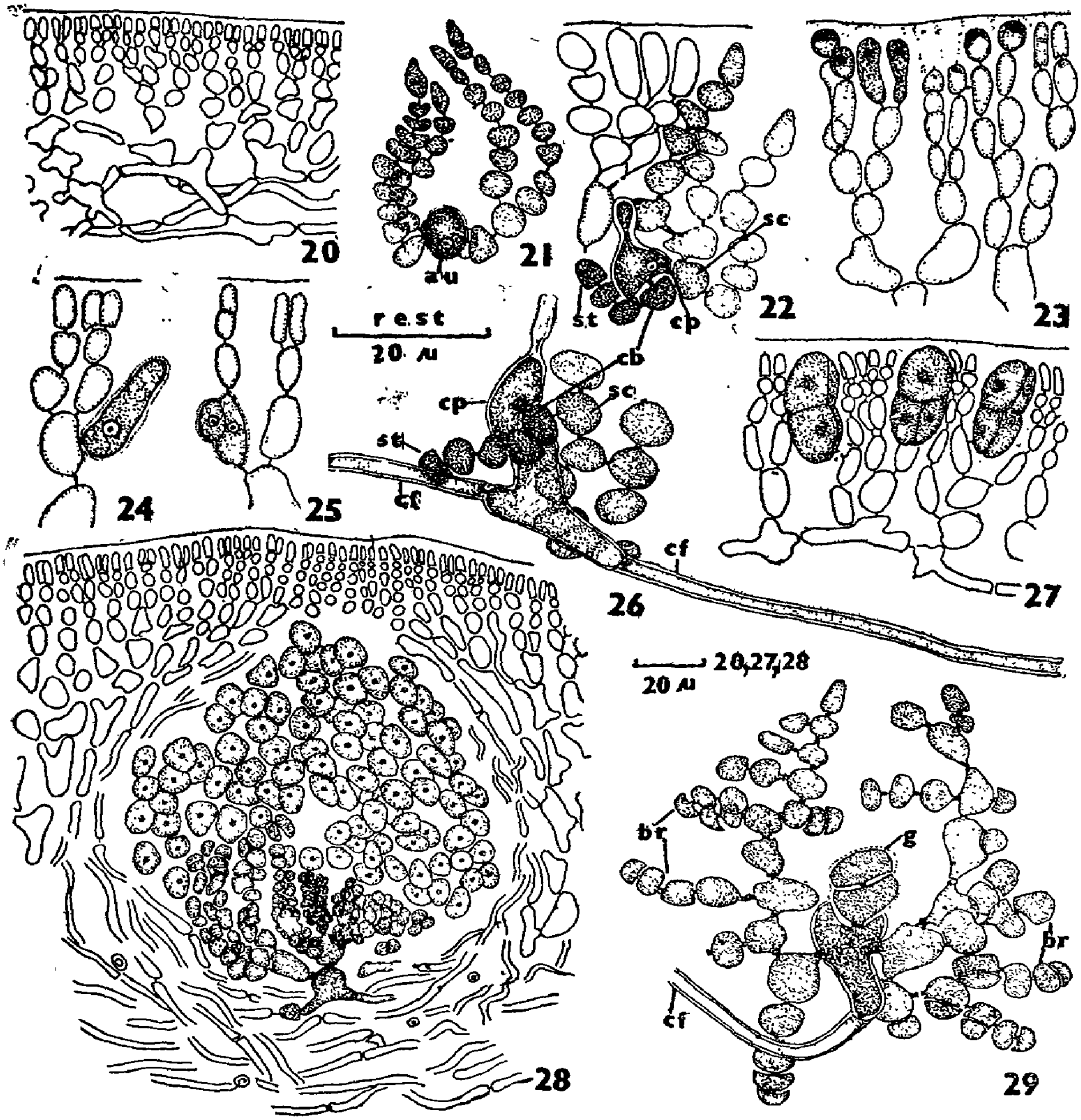
were clearly seen, and it can now be definitely stated that in *G. lithophila* also it is the fertilized carpogonium alone which enlarges and sends out connecting filaments without any fusion with adjacent cells of the carpogonial ampulla. The carpogonial branch-cell and its lateral sterile branch could be clearly distinguished in their entirety in the cases observed (Text-fig. 26, *cb, st.*) The enlarged carpogonium is slightly larger and more irregular than in *G. comorinii,* but it does not attain the size and lobing seen in *G. indica.* Correspondingly, during its development, the cells of the carpogonial ampulla get more impoverished than in *G. comorinii* though not to that extent as in *G. indica* (cf. Text-figs. 5, 15 and 26). The connecting filaments are thick walled, nonseptate and 2·5-4 μ in diameter. They run out in all directions, seeking the auxiliary ampullae and fusing with the auxiliary cells in them. After fusing with one auxiliary cell, in many cases the connecting filament continues its growth, seeking other auxiliary cells and fusing with them.

After fusion with the connecting filament and presumably after receiving a diploid nucleus from it, the auxiliary cells cuts off the gonimoblast initial at the top by a slightly concave wall. This gonimoblast initial, by further development gives rise to the gonimoblast, which is a compact subspherical cluster divided into several gonimolobes. As in the other two previously described species, here also the lower derivative of the auxiliary cell fuses with the adjacent cells of the primary ampullary filament and develops into an irregular fusion-cell which is prominently seen at the base of the ripe cystocarp (Text-fig. 28).

Side by side with gonimoblast development, the erect secondary ampullary branches begin to develop lateral branchlets which form a network more or less completely enveloping the young gonimoblast (Text-fig. 29) (see also Text-figs. 11, 12 in Balakrishnan, 1949). During the later stages of gonimoblast development the cells of this network become impoverished and finally, in the ripe cystocarp, only remnants of the network are seen. As already suggested (Balakrishnan, 1949, p. 208) these branchlets are presumably nutritive in function.

The ripe cystocarps are scattered over the thallus surface. They are subspherical to spherical, 100-200 μ in diameter and are completely immersed in the thallus. They have a single central ostiole through which the ripe carpospores escape (Text-fig. 28). Ripe carpospores are polyhedral with rounded corners to irregular

rounded, 10-15 μ in diameter, uninucleate, and have a number of short ribbon-shaped chromatophores.



TEXT-FIGS. 20–29: *Grateloupia lithophila* Boergs.

20. Section of thallus showing internal organization. 21. Auxiliary ampulla. 22. Young carpogonial ampulla. 23. Section of spermatangial sorus showing the formation of spermatangia. 24. & 25. Early stages in the formation of tetrasporangia. 26. Fertilized carpogonium sending out two connecting filaments; note the carpogonial branch cell (*cb*) and its lateral sterile branch (*st*) which still retain their identity. 27. Section of tetrasporangial plant showing three mature tetrasporangia. 28. Ripe cystocarp in section. 29. Auxiliary ampulla showing an early stage in gonimoblast development and the formation of the nutritive side branchlets.

(*au*: auxiliary cell; *br*: nutritive side branchlets; *cb*: carpogonial branch cell; *cf*: connecting filament; *sp*: carpogonium; *g*: gonimoblast; *sc*: supporting cell; *st*: lateral sterile branch).

Development of tetrasporangia:

The tetrasporangia arise as one-celled secondary lateral branches from cells of the inner cortex. The tetrasporangium initial is cut off by a curved wall from the top portion or occasionally the side of the mother cell (Text-fig. 25) and soon enlarges to form an oblong cylindrical to ellipsoidal tetrasporangium (Text-fig. 24). The tetrasporangial nucleus also enlarges till it is about 5 μ in diameter. Reduction division then takes place in the tetrasporangium (Balakrishnan, 1949, p. 211, Text-fig. 15). After the heterotypic mitosis, cytokinesis takes place, leading to the formation of two uninucleate daughter-protoplasts one above the other. The nucleus in each protoplast divides once again (Mitosis II, see Text-fig. 16 in Balakrishnan, 1949) and four daughter nuclei are organized. A vertical cytoplasmic cleavage takes place in each protoplast resulting in four uninucleate tetraspores. The tetraspores are cruciately or decussately arranged depending on whether the planes of vertical cleavage coincide or are perpendicular to each other (Text-fig. 27). Mature tetrasporangia are ovoid, 20 to 35 μ long and 15-20 μ broad; the ripe tetraspores are densely protoplasmic and contain a number of parietal ribbon-shaped chromatophores (see Text-fig. 18 in Balakrishnan, 1949).

Conclusion

In the ontogeny of the cystocarp, the three species of *Grateloupia, G. comorinii, G. indica* and *G. lithophila,* show very close agreement and are in agreement with that known already for other species of the genus (Berthold, 1884; Kylin, 1930; Balakrishnan, 1949; Kawabata, 1954a; Dawson, 1954).

The carpogonial branches and auxiliary cells are formed separate from one another, remote and borne in distinct ampullary clusters which are special accessory fertile branches. The supporting cell of the carpogonial branch is generally an intercalary cell of the primary ampullary filaments. The carpogonial branch is two-celled. It bears a short lateral sterile branch of 2-6 cells.

Following fertilization, the carpogonium enlarges, and without fusing with any of the cells of the carpogonial ampulla, sends out a number of connecting filaments, each of which seeks out an auxiliary cell and fuses with it. The connecting filaments, after fusion with an auxiliary cell is capable of continuing its growth and fusing with other auxiliary cells,

After fusion with the connecting filament, the auxiliary cell cuts off the gonimoblast initial towards the top. The gonimoblast initial by its activity produces a compact gonimoblast made up of several distinct gonimolobes. During the early stages of gonimoblast development a nutritive basket of interlocked fiilaments is developed from the ampullary branches; this basket is used up during later development and only remnants are left in the mature cystocarps. The ripe cystocarps do not possess a pericarp.

Acknowledgements

The writer is greatly indebted to Prof. M. O. P. Iyengar under whose kind guidance the investigation was carried out and to Prof. T. S. Sadasivan for kind encouragement and advice. To Dr. T. V. Desikachary the writer is grateful for loan of literature and material and much helpful critical discussion. The writer's thanks are also due to Prof. T. S. Mahabalé and the authorities of the University of Poona for kindly granting him study leave during the period of the present investigation.

REFERENCES

Balakrishnan, M. S. (1949) The developmental morphology and cytology of *Grateloupia lithophila* Boergs. *J. Indian bot. Soc.* 28: 205-12.

Berthold, C. (1884) Die Cryptonemiaceen des Golfes von Neapel und den angrenzenden Meeresabschnitte. *Fauna und Flora Golfes von Neapel, Monogr.* 12: 1-27.

Boergesen, F. (1932) Some Indian Rhodophyceae especially from the shores of the Presidency of Bombay, II. *Kew Bull. Misc. Infn.* 3: 113-134.

—— (1938) Contributions to a South Indian Marine Algal Flora III. *J. Indian bot. Soc.* 17: 205-242.

Daines, L. L. (1913) Comparative development of the cystocarps of *Antithamnion* and *Prionitis*. *Univ. Calif. Publ., Bot.* 4: 283-302.

Dawson, E. Y. (1954) Marine Red Algae of Pacific Mexico II. Cryptonemiales (contd.) *Allan Hancock Pacif. Exped.* 17: 241-397.

Drew, K. M. (1954) The organization and interrelationships of the carposporophytes of living Florideae. *Phytomorphology* 4: 55-69.

—— (1957) The Red Algae (1906-1956 and after), *Nature, Lond.,* 180: 112-15.

Fritsch, F. E. (1945) The structure and Reproduction of the Algae. 2: xiv + 1-939, Cambridge.

Kawabata, S. (1954a) On the structure and reproductive organ of *Grateloupia turuturu* Yamada. *Bull. Jap. Soc. Phycol.* 2: 29-32.

—— (1954b) On the structure of the frond, and the reproductive organ of *Pachymeniopsis lanceolata* Yamada (*Aeodes lanceolata* Okamura). *Ibid.* 2: 67-71.

—— (1955) On the structure of the frond, and the reproductive organ of a red alga belonging to the Grateloupiaceae. *Ibid.*, 3: 6-10.

—— (1956) On the structure of the frond, and the reproductive organ of *Cyrtymenia sparsa* Okamura. *Ibid.* 4: 8-13.

—— (1957) On the structure of the frond and the reproductive organ of *Pachymeniopsis Yendoi* Yamada (Syn. *Pachymenia carnosa* Yendo *non* J. Ag.) *Ibid.* 5: 8-13.

Kylin, H. (1930) Über die Entwicklungsgeschichte der Florideen. *Acta Univ. Lund.* N. F. Avd. 2, 26: 1-104.

—— (1932) Die Florideenordnung Gigartinales. *Ibid.* 28: 1-88.

—— (1956) *Die Gattungen der Rhodophyceen*, xv + 672 pp., Gleerup, Lund.

Norris, R. E. (1957) Morphological Studies on the Kallymeniaceae. *Univ. Calif. Publ., Bot.* 28: 251-334.

Okamura, K. (1908) *Icones of Japanese Algae*, 1: 1-258; Tokyo.

—— (1912) *Icones of Japanese Algae*, 2: 1-191, Tokyo.

—— (1913) *Icones of Japanese Algae*, 3: 1-218, Tokyo.

—— (1921) *Icones of Japanese Algae*, 4: 1-205, Tokyo.

Papenfuss, G. F. (1951) Problems in the classification of the marine algae. *Svansk. Bot. Tidsster.* 45: 4-11.

—— (1955) Classification of the Algae, in *A Century of Progress in the Natural Sciences*, 1853-1953. Calif. Acad. Sc. pp. 115-224, San Francisco.

Prakasa Rao, C. S. (1956) The life-history and reproduction of *Polyides Caprinus* (Gunn.) Papenf. *Ann. Bot., Lond.* N. S. 20: 211-30.

Schmitz, C. J. F. (1883) Untersuchungen über die Befruchtung der Florideen. *Sitzb. K. Preuss. Akad. Wiss.* 1883: (1): 215-58.

—— and Hauptfleisch, P. (1897) *Rhodophyceae* in A. Engler und Prantl. *Die Natürlichen Pflanzenfamilien*, 1 (12): 298-544, Leipzig.

Srinivasan, K. S. (1946) Ecology and Seasonal succession of the marine algae at Mahabalipuram (Seven Pagodas), near Madras. *J. Indian bot. Soc.*, M. O. P. Iyengar Comm. Vol. 267-278.

Sjöstedt, L. G. (1926) Floridean Studies, *Acta Univ. Lund*, N.F. Avd. 2, 22: 1-95.

J. Madras Univ., B. XXXI, No. 1, Pages 37-46, 1961.

A Contribution to the Embryology of *Epithema carnosum*

BY

D. PADMANABHAN,

Department of Botany, Presidency College, Madras.

Introduction

(*Received for publication on* 28-1-1961)

ABSTRACT

The sequence of origin of floral envelopes in *Epithema carnosum* is as follows: sepals, stamens, petals, carpels.

Three wall layers are differentiated in the young anther — the endothecium, the middle layer and the tapetum. The tapetal cells become binucleate and behave in a secretory manner. The microspore mother cells undergo division by the furrowing method, resulting in tetrahedral tetrads of microspores. The pollen grains are shed at the two-celled condition.

The ovules are unitegmic and tenuinucellate. The single archesporial cell functions directly as the megaspore mother cell, and the latter forms a linear tetrad of megaspores subsequent to meiosis. The chalazal megaspore develops into an eight nucleate embryo sac of the Polygonum type.

The first division of the endosperm nucleus is followed by the organization of the primary micropylar and chalazal chambers. The micropylar chamber divides vertically to result in two juxtaposed cells which undergo segmentation in a transverse plane. The primary chalazal chamber forms a two nucleate haustorium, while the upper tier of cells derived from the primary micropylar chamber organises the micropylar haustorium with two uninucleate cells.

The seed coat is fomed of an outer layer of thick-walled cells and an inner layer. The thickenings on the radial walls of the outer layer of cells are not uniform and present curious forms in transections.

The earliest contribution to the embryology of the Gesneriaceae is that of Hielscher (1883) who studied the biology and morphology of *Streptocarpus.* Subsequently, the genus *Klugia* was studied by Balicka-Iwanowska (1889) and also by Schnarf (1917). In 1907 Cook's study of *Rhytidophyllum* was published. Two species

of *Ramondia* were investigated by Glisic (1924). All these authors have reported the differentiation of micropylar and chalazal haustoria in the taxa studied by them. *Corytoloma cyclophyllum* (Laurent, 1923) and *Roettlera* (Glišič, 1934) were found to develop a binucleate chalazal haustorium. Considerable variation in the origin of the chalazal haustorium was recorded by Glišič (1928) in *Haberlea rhodopensis*.

Recently Thathachar (1942) published an account of the embryology of *Didymocarpus tomentosa*. This was soon followed by the study on *Rhynchoglossum obliquum* (Thathachar, 1943). Crete's (1942) thesis includes detailed investigations on the embryology and development of endosperm in *Streptocarpus* and *Ramondia*. He reported the absence of chalazal haustorium in an interspecific hybrid of *Streptocarpus*. An account of the development of the endosperm and embryo in *Chirita lavandulacea* (Crété, 1949) and of the embryo sac and endosperm in *Alloplectus sanguineus* (Crété, 1955) have been published recently. The present study deals with the embryology of *Epithema carnosum*.

Flowering material of *Epithema carnosum* was collected from Poringalkuthu rain forests of Kerala State during 1958. The customary methods of infiltration and embedding were followed. Sections were cut at 10-12 microns and stained with iron alum hematoxylin.

Observations

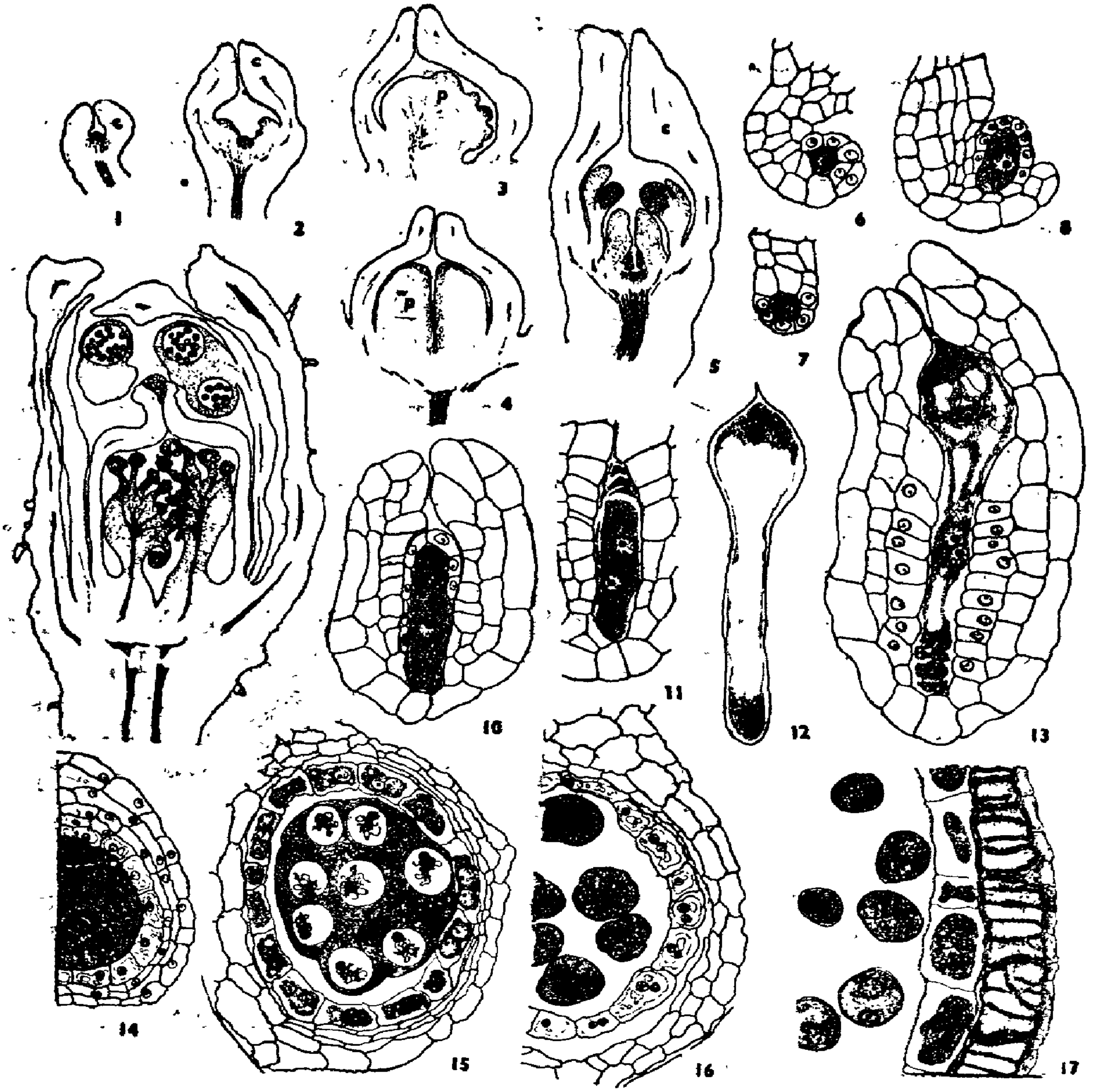
Floral organogenesis :

The earliest primordia to arise on the floral apex are those of the sepals (Text-fig. 1). Following the origin of sepals, the staminal and corolline primordia get differentiated (Text-fig. 2). Subsequently the carpel primordia appear (Text-fig. 5). Two placental ridges differentiate at the base of the carpel primordia (Text-figs. 5, 4). Each of the placental ridges grows initially in the vertical direction. Later, growth is confined to the marginal regions as a result of which a more or less peltate placental ridge gets organized. Numerous ovule primordia appear on the surface of the placenta (Text-figs. 3, 9).

The ovule primordium initially develops into a papillate structure, three cell layers thick (Text-fig. 7). The tip of this structure could be distinguished as the incipient nucellus. Soon a hypodermal cell in this part enlarges and functions as the

archesporium (Text-fig. 7). The archesporial cell functions directly as the megaspore mother cell.

As soon as the hypodermal archesporial cell could be distinguished the ovule primordium begins to assume the anatropous condition (Text-fig. 6). The bending of the primordium is caused by unidirectional cell enlargement and growth at the base (Text-fig. 6). A periclinal division in one of the epidermal cells lying



FIGS. 1-5. Diagrams illustrating floral organogenesis. c-calyx, p-placentum. all X 125; FIGS. 6, 7, 8. Some stages in the development of the ovule. FIGS. 6, 7. X 500. FIG. 8. X 400. FIG. 9. Longitudinal section of the flower. X 100. FIGS 10. L.s. of ovule showing tetrad of megaspores. FIG. 11. Two nucleate embryo sac. FIG. 12. Four nucleate embryo sac; all the nuclei in division. FIG. 13. L.s. ovule showing mature embryo sac. FIGS. 14-17. Stages in the development of the anther.

adjacent to the archesporial cell marks the initiation of the single integument (Text-figs. 6, 8). The rest of the epidermal cells in the apical part of the nucellus divide anticlinally. The megaspore mother cell enlarges about four times its original height (Text-fig. 8). The integument overgrows and envelopes the nucellar tip, the micropyle being organised during the process (Text-fig. 10).

Microsporogenesis :

In the young anther the microspore mother cells are enclosed by three wall layers—the endothecium, the middle layer and the tapetum (Text-fig. 14). The tapetal cells enlarge considerably and exhibit dense staining cytoplasm (Text-fig. 15). With the onset of meiosis in the spore mother cells the tapetal cells become binucleate (Text-fig. 15). At the time of pollen formation, the two nuclei of the tapetal cell fuse to form a dark-staining mass (Text-fig. 17). The tapetal cells ultimately degenerate *in situ*. Hence their behaviour corresponds to the secretory type.

The hypodermal cells of the sporangial wall develop into the endothecium. Thickening strands appear in the walls of these cells (Text-fig. 17). The strands are disposed in radial planes and are often found branched. The epidermal cells remain intact even at the time of dehiscence.

The microspore mother cells enlarge slightly before embarking on the meiotic divisions. At the close of meiosis II, the four resulting nuclei are aligned in a tetrahedral manner. Clevage furrows are initiated in the cytoplasm and the cell gets partitioned simultaneously into four microspores arranged in a tetrahedral form (Text-fig. 16). Soon after, the microspores separate from the tetrahedral configuration and increase slightly in size. The microspore nucleus undergoes division to form a smaller generative cell and a larger vegetative cell. The generative cell soon comes to lie without any definite position within the cytoplasm of the vegetative cell. In this condition the pollen grains are shed.

Megasporogenesis and female gametophyte :

The megaspore mother cell enlarges prior to division. At the close of meiosis a linear tetrad of megaspores is formed. The chalazal megaspore enlarges and matures into the gametophyte while the three micropylar ones get disorganized and appear as black masses (Text-figs. 10, 11). Prior to the first division of the functional megaspore the nucellar epidermis gets disorganized

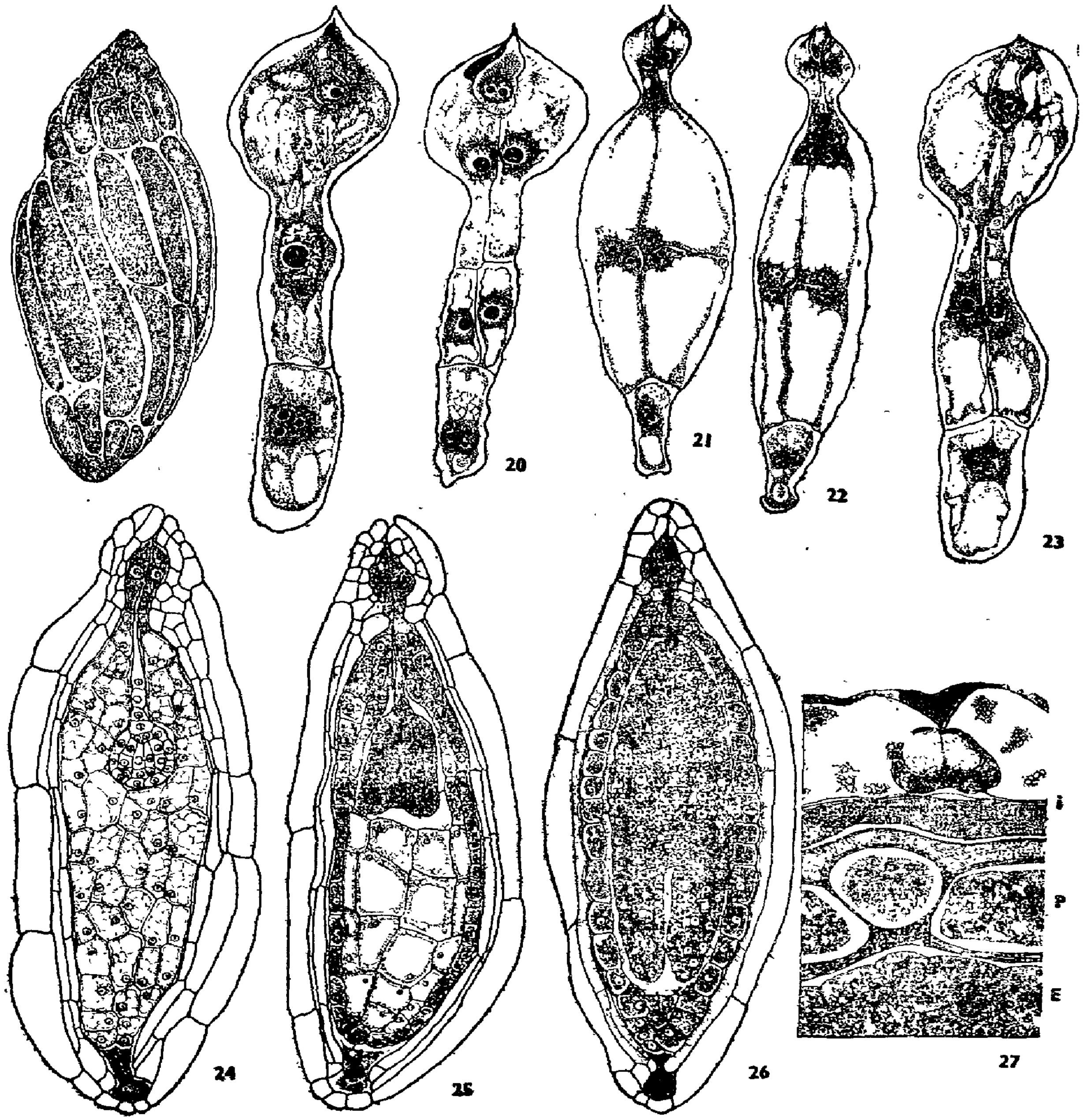
(Text-fig. 11). After the first division of the megaspore mother cell, the daughter nuclei migrate to the poles, a large central vacuole appears and the micropylar end of the embryo sac grows into a bulbous structure (Text-fig. 12). The division of the two daughter nuclei results in four nuclei, which in turn soon embark on mitosis (Text-fig. 12). The eight nuclei of the mature embryo sac organize an egg apparatus, two polars and three antipodal cells. The egg apparatus consists of a pair of synergids each with a basal vacuole and a beak-like tip, and an egg cell. The antipodals are arranged in a linear fashion at the chalazal end while the egg apparatus is situated in the bulbous micropylar end. The polars lie juxtaposed in the middle part of the embryo sac (Text-fig. 13).

The integumentary cells bordering upon the lower tubular part of the embryo sac elongate in the radial direction and assume the characteristics of endothelium.

Development of endosperm:

Soon after double fertilization the synergids are disorganized and the triple fusion nucleus undergoes division followed by the deposition of a transverse wall which separates the micropylar and chalazal chambers (Text-fig. 19). The chalazal chamber functions as the chalazal haustorium and the antipodals included in it degenerate and are completely absorbed. Even at early stages of endosperm development the chalazal chamber becomes binucleate due to one free nuclear division (Text-figs. 20-22). The primary micropylar chamber is partitioned vertically followed by transverse cytokinesis (Text-figs. 23, 20). The resulting four cells are arranged in two superposed tiers of two cells each. The micropylar tier constitutes the micropylar haustorium which remains two-celled even at maturity. The subjacent tier augments the formation of endosperm proper. Divisions in this tier initially occur in predominantly in transverse planes (Text-figs. 21, 22). Hand in hand with the early development of the haustoria the central part of the embryo sac elongates considerably accompanied by enlargement in diameter. The zygotic tube becomes situated in between the two cells of the micropylar haustorium. The cells of the incipient endosperm proper (Text-figs. 21, 22) appear highly vacuolated and elongated in the direction of the long axis of the embryo sac. Further divisions occur in transverse and longitudinal planes and a mass of endosperm cells is formed (Text-fig. 24). By this time the apical pole of the developing proembryo

gets embedded in the cellular part of the endosperm proper (Text-fig. 24). Internal differentiation in the endosperm mass occurs subsequently. Three regions could be recognized: the outermost layer of deep staining cells, the middle layer of slightly vacuolate narrow cells and innermost group of highly vacuolate and large cells (Text-fig. 25). The embryo consumes all the endosperm cells, except the outermost layer and small groups of



FIGS. 10-17. X 400.

FIG. 18. Surface view of mature seed. X 200. FIGS. 19-23. Stages in the early development of endosperm. FIGS. 19, 20. X 800; FIGS. 21, 22. X 320. FIGS. 24-26. L.s. through ovule showing the late development of endosperm and haustoria. X 160. FIG. 27. T.s. of seed, a portion of seed coat enlarged. i-integument, P-persisting layer of endosperm cells, E-embryo.

cells at the micropylar and chalazal ends which persist in the seed (Text-figs. 25-27).

The chalazal haustorium exhibits little activity except for the slight enlargement at the basal end (Text-figs. 22, 23, 25, 26). With the maturation of the seed the chalazal haustorium acquires a dark staining mass of cytoplasm, and its two nuclei fuse to form a black mass (Text-fig. 26). Similar developments occur in the micropylar haustorium excepting nuclear fusion.

The seed:

At the time of fertilization, the integument is made up of about three to four layers of cells. The innermost layer functioning as the endothelium gets stretched during the early development of the endosperm (Text-fig. 24). The cells of the outermost layer enlarge considerably and their radial walls get thickened in a peculiar fashion. In transsections of the seed the thickening band on the proximal half-length of the radial wall appears in the form of the letter W with the two lateral arms bent inwards and touching each other. In the upper half-length, the thickening appear V-shaped (Text-fig. 27). The characteristic long spirally twisted ridges seen on the surface of the seed (Text-fig. 18) are the loci of this curious type of wall thickening. The cells of the endothelium filled with pale yellow contents get adpressed to the outer layer, the middle layers being completely absorbed. Wall thickening occurs in the persisting endosperm cells. Large dark staining granules of proteinaceous nature are densely distributed in the persisting endosperm cells. It is of interest to note that cells of the embryo also get filled with such granules at mature stages of the seed (Text-fig. 27).

The embryo:

The embryo of *Epithema carnosum* follows the Onagrad type of development. A detailed account of embryogenesis will be published separately.

Discussion

Microsporogenesis

The structure of the anther in *Epithema carnosum* is similar to that reported for *Didymocarpus tomentosa* (Thathachar, 1942). Three wall layers are differentiated in both species—the endothecium, one middle layer and the tapetum. The middle layer gets obliterated at late stages. The behaviour of the anther tapetum

corresponds to the secretory type. The tapetal cells become binucleate in *Epithema* whereas in *Didymocarpus* as many as five free nuclei are formed. The free nuclei formed in each tapetal cell undergo fusion at late stages resulting in a dark staining mass. This again is a common feature in *Epithema* and *Didymocarpus* Cytokinesis in the pollen mother cells occurs by simultaneous deepening of peripheral furrows. The pollen grains are shed at the two-celled stage in *Epithema carnosum* (present work), *Didymocarpus tomentosa* (Thathachar, 1942) and *Gloxinia hybrida* (Elfving, 1879).

Megasporogenesis and female gametophyte

The investigated members of the Gesneriaceae exhibit tenuinucellate unitegmic ovules, hypodermal archesporium and linear tetrads of megaspores. The embryo sac of all species studied exhibits a bladder like micropylar part and a tubular chalazal portion. Development of integumentary tapetum is confined to that part of the ovule bearing the tubular part of the embryo sac. While the antipodals and Synergids disorganise after fertilization in all the species studied, a peculiar case of persistence of one synergid has been recorded in *Rhynchoglossum obliquum* (Thathachar, 1943). Extramicropylar development of the embryo sac was recorded in *Roettlera* (Glišič, 1934) and in *Monophyllaea horsefieldii* (Oehlkers, 1923). In *Didymocarpus tomentosa* (Thathachar, 1942) the embryo sac exhibits no tendencies towards extra-micropylar development even though the endosperm haustorial cells grow out of the micropyle. In *Epithema carnosum*, neither the embryo sac nor the endorsperm haustoria show extra-micropylar developments.

Endosperm

As summarized by Thathachar (1943) and Crete (1955) the development of endosperm in the Gesneriaceae falls under two distinct types. In one type, the division in the primary micropylar chamber is vertical while in the other a transverse wall is laid down resulting in a row of three endosperm cells, including the primary chalazal chamber. *Corytoloma cyclophyllum* (Laurent, 1923), *Haberlea rhodopensis* (Glisic, 1928), *Roettlera* (Glisic, 1934), *Didymocarpus tomentosa* (Thathachar, 1942) and *Epithema carnosum* (present study) belong to the first type, whereas *Klugia zeylanica* (Schnarf, 1921) *Ramondia Nathaliae* and *Ramondia serbica* (Glišič, 1924) fall under the second type. In all species investigated, the micropylar haustorium is constituted of two uni-

nucleate cells. Only in *Didymocarpus tomentosa* an extramicropylar development of the haustorial cells has been recorded.

The chalazal haustorium is formed of two uninucleate cells in *Haberlea rhodopensis* (Glišič, 1928). Two-nucleate single celled chalazal haustoria have been recorded in *Didymocarpus tomentosa* (Thathachar, 1942), *Roettlera* (Glišič, 1934), *Haberlea* (Glišič, 1928), *Alloplectus sanguineus* (Crété, 1955), *Chirita lavandulacea* (Crété, 1949) and *Epithema carnosum* (present work). Uninucleate condition has been recorded in *Rhynchoglossum* (Thathachar, 1943) *Klugia zeylanica* (Schnarf, 1921) and two species of *Ramondia* (Glišič, 1924). No chalazal haustorium is formed in a hybrid *Streptocarpus* (Crété, 1942).

Acknowledgement

I am profoundly thankful to Prof. B. G. L. Swamy, for guidance and encouragement.

REFERENCES

Balicka-Iwanowska, G. (1899) "Contribution a l'étude du sac embryonnaire chez certaines Gametopetales". *Flora* **86**: 47-71.

Cook, M. Th. (1907) The embryology of *Rhytidophyllum crenulatum*. *Bull Torrey bot. Cl.* **34**: p. 179.

Crété,. P. (1942) *Recherches histologiques et physiologiques sur l'embryologie des Labiatiflores.* Thèse Doct. ès Sc. Paris.

—— (1949) Recherches embryologiques sur les Gesneracees. Developpement de l'albumen et de l'embryon chez le *Chirita lavandulacea* Staf. *Bull. Soc. bot. Fr.* **96**: 234-235.

—— (1955) L'origine du sac embryonnaire et la developpement de l'albumen chez l'*Alloplectus sanguineus* Mart. (Gesnèracées). *Ibid.* **102**: 205-208.

—— (1955) L'application de certaines données embryologiques à systématique des Orobanchacées et de quelques familles voisines. *Phytomorphology* **5**: 422-435.

Elfving, F. (1879) Studuien über die Pollenkörner der Angiospermen. *Jenaische Z. Med. Nat.* **13**: 1-28.

Glišic, L. M. (1924) Development of the female X-generation and embryo in *Ramondia*. Diss. Philos. Fak. Beogard.

Glišic, L. M. (1928) Development of the female gametophyte and endosperm in *Haberlea rhodopensis* Friv. *Bull. Inst. bot. Univ. Belgrade*, 1: 1-13.

—— (1934) Zur Kenntnis der Samenentwickelung der Gesneriaceen. Über die Endosperm und Haustorienbildung von *Roettlera*. *Ibid.* 3: 94-111.

Hielscher, T. (1883) Anatomie und Biologie der Gattung *Streptocarpus*. *Beitr. Biol. Pfl.* 3: 1-24.

Laurent, V. (1923) Zur Entwicklungsgeschichte von *Corytoloma cyclophyllum* Dusen. *Svensk bot. Tidskr.* 17: 164-174.

Oehlkers, Fr. (1923) Entwicklungsgeschichte von *Monophylleoa Horsfieldii*. *Beih. z. bot. Zentrbl.* 39: 1 p. 128.

Schnarf, K. (1921) Kleine Beiträge zur Kenntnis der Entwicklungsgeschichte der Angiospermen. II. *Klugia zeylanica*. (R. Brown) Gardn. *Öst. bot. Zeit.* 70: 255-261.

Thathachar, T. (1942) Studies in the Gesneriaceae. Gametogenesis and embryogeny of *Didymocarpus tomentosa* Wt. *J. Indian bot. Soc.* 21: 185-194.

—— (1943) Studies in Gesneriaceae. Development of seed of *Rhynchoglossuum obliquum* Bl. *J. Indian bot. Soc.* 22: 151-157.

J. Madras Univ., B. XXXI, No. 1, Pages 47-51, 1961.

Embryogenesis in *Dorstenia indica*

BY

D. PADMANABHAN

Department of Botany, Presidency College, Madras

(Received for publication on 2-2-1961)

ABSTRACT

The development of embryo in *Dorstenia indica* conforms to the Urtica variation of Asterad type. However, there are minor differences from *Urtica pilulifera*. As far as embryogenesis is concerned *Dorstenia* falls in line with other members of the Moraceae so far investigated.

Introduction

The existing literature on embryogenesis in the Moraceae is too meagre to permit any generalization. Four species belonging to four genera have been studied: *Artocarpus integrifolia* (Subba Rao, 1940), *Streblus asper* (Anantasamy Rau, 1942), *Ficus carica* (Condit, 1931), *Morus* sp. (Singh, 1954). Recently Johri and Konar (1956) have included a brief account of embryogeness in their study of *Ficus religiosa*. The present paper deals with embryogenesis in *Dorstenia indica*. Fruiting specimens of *Dorstenia indica* were collected at Kodaikanal, S. India, during October, 1956.

Observations

The zygote undergoes division by a transverse wall (Text-figs. 1, 2). The terminal cell *ca* is partitioned vertically to form two juxtaposed cells while the basal cell *cb* divides transversely to form two superposed cells *m* and *ci* (Text-figs. 3, 4). Thus a T-shaped proembryonal tetrad is erected at the second cell generation.

The next division in the terminal tier of the four-celled proembryo takes place in the vertical plane but at right angles to the first formed partition and the resulting four cells form the quadrants *q* (Text-fig. 5). Meanwhile cell *m* divides vertically and *ci* is segmented transversely to form *n* and *n'* (Text-fig. 5).

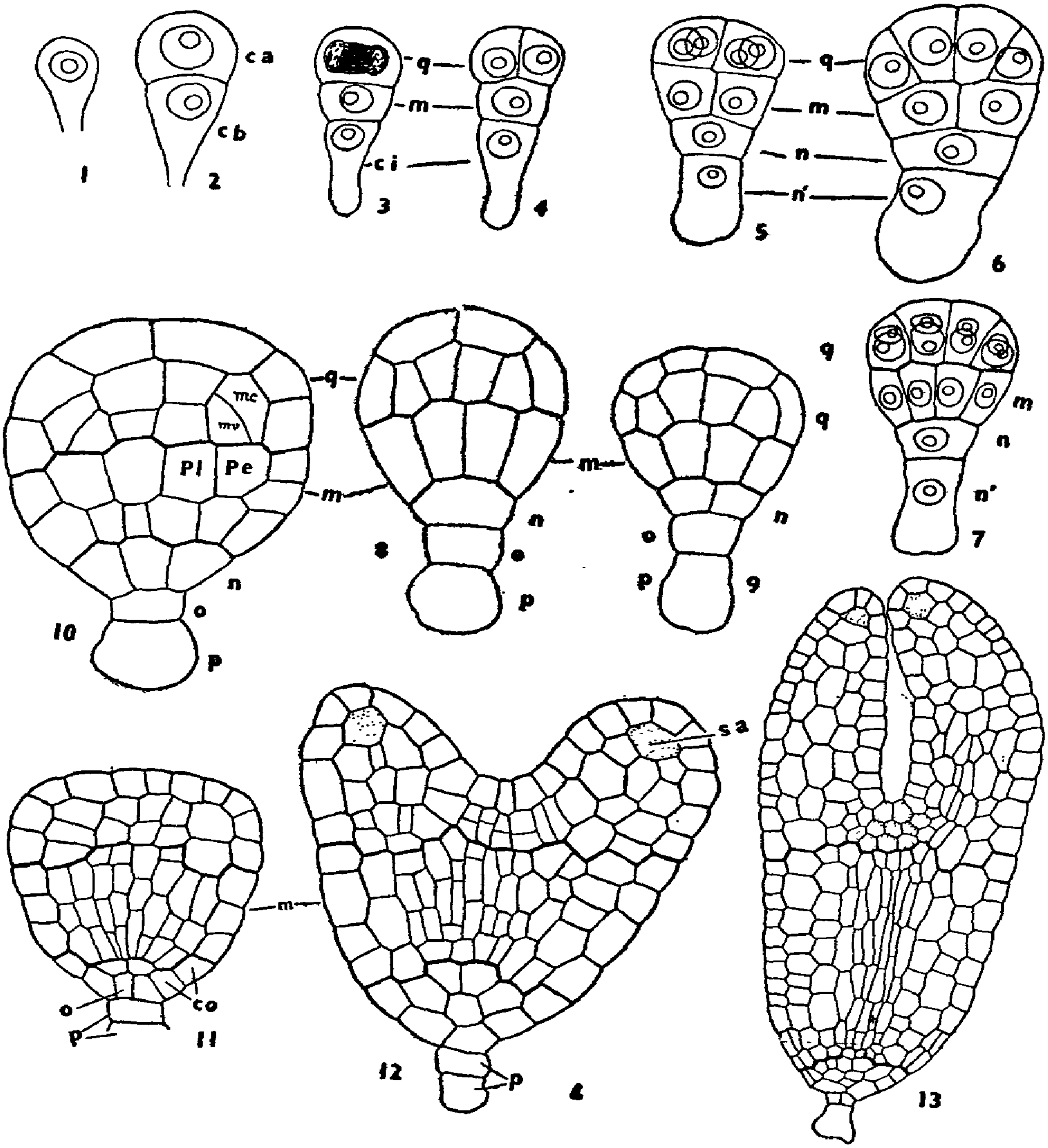
The octants are originated by the deposition of obliquely vertical walls in the quadrant cells (Text-fig. 6). At this stage tier *q* consists of four circumaxial cells surrounded by a corresponding number of peripheral cells. Thus all the eight cells are disposed in a single tier *q*. Subsequent development in the proembryo is associated with initiation of histogens.

Of the eight octant cells, the circumaxial ones undergo a transverse division, the upper derivatives forming the initials of dermatogen while oblique walls in the peripheral cells isolate the epidermal initials (Text-fig. 8). The two juxtaposed cells composing tier *m* undergo vertical divisions forming a group of four circumaxial cells. Initiation of histogens in this tier takes place by the deposition of vertical walls in the concerned cells in sueh a manner that four peripheral epidermal initials are isolated (Text-fig. 7). At this stage the tier *m* is composed of four axial cells surrounded by four peripheral cells. The axial cells divide again in the vertical plane and the derivatives adjoining the dermatogen form the cortical initials (pe) while the rest function as the plerome initials (pl) (Text-fig. 10). It is of interest to note that generally initiation of histogenic layers takes place earlier in the tier *m* than in the terminal tier *q* (Text-fig. 7).

The cell *n* undergoes cytokinesis in the vertical plane while *n'* is segmented transversely to form *o* and *p* (Text-fig. 9). Subsequently the cell *p* undergoes a transverse division. The lower derivative becomes swollen and forms a suspensor along with the upper derivative (Text-figs. 10, 11).

Subsequent development leads to the attainment of the late globular stage. Further cell divisions in the tier *q* are associated with the initiation of cotyledons. After the isolation of the epidermal initials the inner derivatives of the octant cells (Text-figs. 8, 9) undergo divisions virtually in the same plane as that of their mother cell (Text-fig. 10). Thus the circumaxial cells undergo divisions in a transverse plane while the peripheral cells are segmented in an oblique plane. Of the derivatives of the peripheral cells two inner ones (*mv* in Text-fig. 10) situated on opposite loci (with reference to the transverse topography of the embryo) are concerned with the initiation of the inner core of the cotyledons while the outer ones (*mc*) take part in the construction of the cortical regions (Text-fig. 10). Further divisions in these cells results in the initial heightening of the cotyledon primordium (Text-figs. 11, 12). When the cotyledons begin to

emerge, a distinct sub-apical-initial cell is organized by one of the derivatives of *mc* (Text-figs. 10-12). The sub-apical-initial cuts off derivatives in the periclinal plane. Anticlinal divisions in the resulting cells are associated with the initiation of the histogenic layers of the cotyledon primordium. An inner core of procambium differentiates in the cotyledon primordium in continuation of the strands formed in the tier *m* (Text-fig. 13).



FIGS. 1-13. See text for explanation.

FIGS. 1-10. X 625. FIGS. 11-13. X 400.

Transverse divisions predominate in the cortical and plerome initials of the tier *m*, particularly at early stages of the globular embryo. Subsequently vertical divisions become frequent. It is of interest to note that while overall cell enlargement is characteristic of the cortical region, cell elongation in the vertical direction is distinctive of the plerome. This is one of the features of *differentiation* within the axial part of the embryo.

A vertical division in the cell *n* resulting in two juxtaposed cells (Text-fig. 9) occurs at early stages. The next division also occurs in the same plane leading to the formation of four cells (Fig 10). The initials of the root cortex (as also part of the root cap, *co* in Text-fig. 11) are isolated by oblique divisions of the cells composing this tier. The derivatives of tier *o* contribute to the central part of the root cap (Text-fig. 11). The plerome initials of the root are derived from the lowermost derivatives of the tier *m*.

At the shedding stage of seed the embryo possesses two well-defined cotyledons each with an axial core of procambium in continuity with the strands in the hypocotyl. The root meristem is well defined. The region in between the cotyledons (the epicotyl apex) exhibits one layer of epidermal cells over a core of three subjacent layers (Text-fig. 13) which are derived by transverse divisions in the inner derivatives of the axial octant cells.

Discussion

Johansen (1950) assigned the development of embryo in the following species to the Asterad type: *Artocarpus integrifolia* (Subba Rao, 1940), *Streblus asper* (Anantaswamy Rau, 1942), *Ficus carica* (Condit, 1931). However, he expressed doubt over the reports on *Ficus carica.* Since the proembryo of this species is stated to be a linear tetrad assignment to the Asterad type is not tenable. On the other hand Condit states that later development resembles that in *Urtica pilulifera* (Soueges, 1921). A similar statement has been made by Subba Rao (1940) in regard to *Artocarpus integrifolia* as also by Singh (1954) on *Morus.* Eventhough Johri and Konar (1956) are not explicit about the type of embryo development in *Ficus religiosa,* their illustrations (their text-figures 66-69) are definitely in favour of assigning this species to the Asterad type. The present investigation indicates that *Dorstenia indica* falls in line with the other members of the family. The Moraceae, therefore, seem to be characterised by the Asterad type of embryogenesis.

Regarding the assignment of the embryogenic type in Moraceae to definite variations Johansen (1950) states that it is probable that *Artocarpus integrifolia* conforms to the Urtica variation. In the case of *Streblus asper* as stated by Johansen, the structure of the indistinct suspensor precludes assigning this species to the Urtica variation. *Dorstenia indica* follows the Urtica variation closely eventhough there are some differnces in the post-octant developments. In *Urtica pilulifera* two subepidermal layers are initiated in the cotyledonary region whereas in *Dorstenia* the prevailing condition indicates no large departure from the typical Asterad type.

Acknowledgement

I am deeply indebted to Prof. B. G. L. Swamy, for guidance and encouragement.

REFERENCES

Anantaswamy Rau, M. (1942) Development of the embryo sac and embryo in *Streblus asper* Lour. *J. Mysore Univ.* **2**: 109-114.

Condit, I. J. (1931) The structuure and development of flowers in *Ficus carica* L. Diss. Stanford Univ. (Quoted from Johansen, 1950).

Johansen, D.A. (1950) *Plant Embryology.* Chronica Botanica. Waltham, Mass.

Johri, B. M. and R. N. Konar. (1956) The floral morphology and embryology of *Ficus religiosa. Phytomorphology* **6**: 97-111.

Singh, S. P. (1954) Morphological studies in *Morus* sp. *Agra. Univ. J. Res. (Sci.).* **3**: 323-341.

Soueges, R. (1921) Developpement de l'embryon chez l'*Urtica pilulifera. Bull. Soc. bot. Fr.* **68**: 172-188, 280-294.

Subba Rao, A. M. (1940) Cytology and embryogeny in *Artocarpus integrefolia* Linn. *J. Mysore Univ.* **2**: B 63-73.

J. Madras Univ., B. XXXI, No. 1, Pages 53-58, 1961.

Studies on Seeds with Ruminate Endosperm

1. Morphology of ruminating tissue in *Myristica fragrans.*

BY

K. PERIASAMY,

Department of Botany, Presidency College, Madras.

(Received for publication on 27-2-1961)

ABSTRACT

The seed of *Myristica fragrans* Houtt., is the classical example that illustrates the character of ruminate endosperm. Its seed development is of the most pronounced massive-chalazal type and the tissue that gives rise to the rumination ingrowths belong wholly to the chalaza. Neither the inner nor the outer integument takes part-in the production of rumination.

Introduction

The ruminate endosperm is a peculiar feature met with in certain members of the higher vascular plants, and the seed of *Myristica fragrans* Houtt., is the classical example that illustrates the phenomenon of rumination. Indeed, it is to explain the characteristic appearance of its endosperm in the mature seed, that the term ruminate endosperm appears to have been first coined. This is indicated by the meaning given by Asa Gray (1879) to the word *ruminate*: "Ruminated — as if chewed; said of the albumen of a nutmeg". A clear understanding of the phenomenon of rumination in this plant is, therefore, necessary.

Seed Development

The investigation of Voigt (1888) on the phenomenon of rumination, gives a detailed account of the post-fertilization development of *Myristica fragrans.* The development of rumination and the structure of the mature seed coat have been clearly dsecribd by him. While, therefore, reference may be made to Voigt (1888) for a detailed account, a brief summary of the seed development in so far as it concerns the phenomenon of rumination, may be given here, based on the description of Voigt and on personal observations.

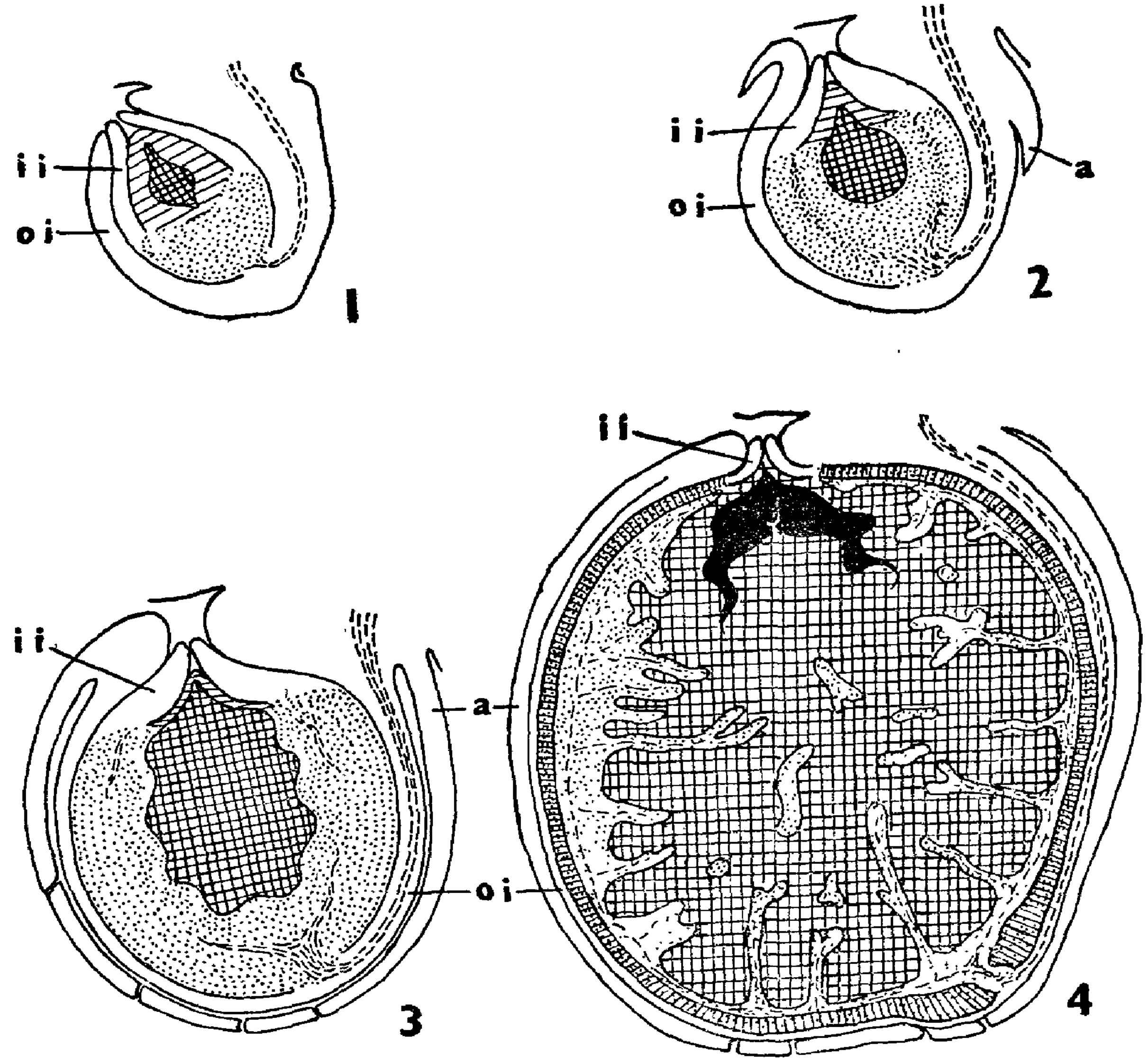
The mature ovule (Text-fig. 1), is anatropous, bitegmic, crassinucellate, and attached basally in the monocarpellary ovary. The vascular strand ends at the chalaza. The micropyle is formed by the inner integument alone. The inner integument is inserted at half the height of the nucellus and does not arise from the basal part from where the outer integument starts. Therefore, a considerable portion of the 'nucellus' below the place of insertion of the inner integument is directly in contact with the outer integument. The outer integument lies loosely around the inner integument, above the level of the latter's insertion, but the inner integument in tightly adpressed to the nucellus.

After fertilization, the entire outer integument as well as the tissues that lie inside it below the level of insertion of the inner integument become actively meristematic and grow rapidly. The inner integument and the nucellus surrounded by it, do not exhibit much meristematic activity and remain arrested more or less in the same state in which they were before fertilization. As a result of this differential growth, the inner integument becomes pushed more and more towards the apex of the enlarging seed (Text-fig. 2).

The nuclear endosperm first consumes the nucellus inside the inner integumuent and then extends lower down into the central part of the tissue that is produced by post-fertilization meristematic activity. The peripheral tissue that surrounds the endosperm below the base of the inner integument, produces, by meristematic activity, finger-like ingrowths which push in the nuclear endosperm and make its surface wavy and irregular (Text-fig. 3). The ingrowths are not absorbed by the endosperm and, therefore, continue to push deeper and deeper into the endosperm until the seed almost attains its mature size. Even after the endosperm becomes cellular, the ingrowths are left intact to constitute the characteristic rumination processes of the mature seed (Text-fig. 4). Neither the inner integument nor the outer one takes part in the production of rumination ingrowths; the inner integument becomes pushed to an insignificant portion around the micropylar part at the apex and the outer integument forms a covering outside the ruminating tissue (Text-fig. 4).

The vascular strand at the chalaza undergoes extensive branching and the branches ramify in the periphery of the tissue that produces the rumination ingrowths. The ingrowths themselves are formed along the lines of vascular ramification and each

ingrowth gets supplied with a radial branch from the peripheral network (Text-fig. 4). The vascular branches extend up to the base of the inner integument but do not actually enter into it.



TEXT-FIGS. 1-4. *Myristica fragrans* (after Voigt, 1888). FIG. 1—Median L.s. of ovule at the time of fertilization. FIGS. 2, 3—Median L.s. of two stages of young developing seed after fertilization. FIG. 4—Median L.s. of mature seed. (chalaza stippled; embryo sac and endosperm cross hatched; nucellus striped).

a—aril; *ii*—inner integument; *oi*—outer integument.

Morphology of the Ruminating Tissue

It may be seen from the above account that the ingrowths of rumination develop from the tissue that arises in basal continuation of the nucellus and the base of the inner integument. Regarding the morphological nature of the tissue, conflicting views have been expressed. Voigt (1888) assumes the tissue

to be a portion of the nucellus itself and designates it as the 'lower nucellar portion' in contrast to the 'upper nucellar portion' which is surrounded by the inner integument in the ovule and which becomes completely replaced by the endosperm in the mature seed. Netolitzky (1926) dos not endorse the view of Voigt (1888), but considers the tissue as belonging to the chalaza morphologically. Sastri (1955), however, considers the tissue as the inner integument, evidently because it occupies a position inside the outer integument and also because it develops below the base of the inner integument of the ovule. The following points taken in conjunction with the observations of Netolitzky (1926) and Voigt (1888), point to the correctness of the view expressed by Netolitzky.

1. Since, in bitegmic ovules, the chalaza in the tissue situated below the place of origin of the integuments (Periasamy, 1961), the tissue below the place of insertion of the inner integument must be looked upon as belonging to the chalaza and not to the nucellus.

2. There is no clearly established instance of a well defined vascular strand within the nucellus in angiosperms. (Only the occurrence of isolated tracheids in the nucellus, without connection to the main vascular supply to the ovule has been reported in certain instances (Bensen, 1894; Frye, 1902; Bensen et al, 1906). Of the other reports of vasculature in the nucellus (Guerin, 1915; Orr, 1921a, b; Grove, 1941; Earle, 1938; Landes, 1946; Fagerlind, 1947), some have been contradicted (Mauritzon, 1939; Fuchs, 1938; Kausik, 1940; Singh, 1959). In other cases also it is probable that the chalazal vascular supply is mistaken to be that of the nucellus, because of failure to identify chalazal hypertrophy).

3. In all cases where there is chalazal growth after fertilization, the entire chalazal region is supplied with vasculature (Periasamy, 1959; Periasamy and Swamy, 1961). The ruminating tissue in *Myristica fragrans* is supplied with a peripheral network of vasculature from which radial branches are given off to the ingrowths.

4. The ruminating tissue becomes histologically quite different from the nucellus, even during the early stages of development. The nucellus is replaced by the endosperm while the ruminating tissue is left intact.

5. The tissue of the inner integument can be seen as an insignificant, but clearly separate tissue at the extreme apex of

the mature seed, but no rumination ingrowths arise from this region. Histologically too, the persisting part of the inner integument is quite different from the ruminating tissue.

Sastri (1955) appears to misinterpret the structure of the mature seed coat in *Myristica fragrans.* The single layer of radially elongated and thick-walled cells which forms the mechanical tissue of the seed coat, and which is designated as the 'Hauptpalisadenlage' by Voigt (1888), is interpreted as consisting of several layers by Sastri. This is evidently because, on account of the small cross-sectional area of the cells of this layer, it presents the appearance of being many layered in sections even slightly oblique to the radial plane in which the cells are elongated.

Periasamy (1961) has distinguished three types of activities in regard to the behaviour of the chalaza after fertilization, as normal-chalazal, perichalazal, and massive-chalazal. Seed development in *Myristica fragrans* is of the massive-chalazal type and represents perhaps the most pronounced expression of this feature among the bitegmic ovules. Even though the ovules are bitegmic, none of the integuments take part in the production of rumination which becomes wholly a function of the hypertrophied chalaza. Further, it is the ruminating tissue that constitutes the most prominent part of the seed coat around the endosperm in the mature seed.

REFERENCES

Bensen, M. (1894) Contribution to the embryology of the Amentiferae 1. *Trans. Linn. Soc. Lond.* II. *(Bot.)* 3: 409-24.

Bensen, M., *et al.* (1906) Contribution to the embryology of the Amentiferae II. *Carpinus betulus. Trans. Linn. Soc. Lond.* II. *(Bot.)* 7: 37-44.

Earle, T. T. (1938) Embryology of certain Ranales. *Bot. Gaz.* **100:** 257-75.

Fagerlind, F. (1947) Gynoecium morphologische und embryologische Studien in der Familie Olacaceae. *Bot. Notiser* **1947**: 207-30.

Frye, T. C. (1902) A morphological study of certain Asclepiadaceae. *Bot. Gaz.* **34**: 389-413.

*Fuchs, A. (1938) Beiträge zur Embryologie der Thymelaeaceae. *Österr. bot. Z.* **87**: 1-41.

Gray, A. (1879) Structural Botany, or Organography on the basis of Morphology: New York.

Grove, A. R. (1941) Morphological study of *Agave lechuguilla*. *Bot. Gaz.* 103: 354-65.

*Guérin, P. (1915) Reliquiae Treubinae. I. Recherches sur la structure anatomique de l' ovule et de la graine des Thyméléacées. *Ann. Jard. bot. Buitenz.* 14: 1-35.

Kausik, S. B. (1940) Structure and development of the ovule and embryo sac of *Lasiosiphon eriocephalus* Dcne. *Proc. natl. Inst. Sci. India* 6: 117-32.

Landes, M. (1946) Seed development in *Acalypha rhomboidea* and some other Euphorbiaceae. *Amer. J. Bot.* 33: 562-68.

Mauritzon, J. (1939) Contributions to the embryology of the orders Rosales and Myrtales. *Lunds Univ. Asskr. N.F. Avd. 2.* 35: 1-120.

Netolitzky, F. (1926) *Anatomie der Angiospermen Samen. Handbuch der Pflanzenanatomie*, Berlin.

*Orr, M. Y. (1921a) The occurrence of tracheids in the nucellus of *Steriphoma cleomides* Spreng. *Notes Roy. bot. Gard. Edinb.* 12: 241-42.

*—— (1921b) The occurrence of a tracheal tissue enveloping the embryo in certain Capparidaceae. *Notes Roy. bot. Gard. Edinb.* 12: 249-57.

Periasamy, K. (1959) *Sutdies on seeds with ruminate endosperm.* Ph.D. thesis, Madras Univ.

—— (1961) The ruminate endosperm—1. Development and types of rumination. *Phytomorphology* (in press).

—— & Swamy, B. G. L. (1961) Studies in the Annonaceae—2. Development of the ovule and seed in *Cananga odorata* and *Miliusa wightiana* *J. Inlian bot. Soc.* (in press).

Sastri, R. L. N. (1955) Structure and development of nutmeg seed. *Curr. Sci.* 24: 172-73.

Singh, S. P. (1959) Structure and development of seeds in *Euphorbia geniculata* Orteg. *J. Indian bot. Soc.* 38: 103-8.

Voigt, A. (1888) Untersuchungen über Bau und Entwicklung von Samen mit ruminertum Endosperm aus den Familien der Palmen, Myristicaceen und Annonaceen. *Ann. Jard. bot. Buitenz.* 7: 150-90.

*Not seen in the original.

J. Madras Univ., B. XXXI, No. 1, Pages 59-62, 1961.

Further Additions to the List of *Mononchus* from Madras City*

BY

S. KANNAN

Department of Zoology, Presidency College, Madras

(Received for publication on 7-1-61)

ABSTRACT

Three more species of the soil nematodes of the genus *Mononchus* Bastian 1865 are described in this paper. Of these one is a new species another is a new variety.

1. *Mononchus* (*Mylonchulus*) *madrasi sp.nov.* (Text-fig. 1.)

Total 78. Female 32; Juveniles 46.

Measurements: L = 0·947 mm. a = 16·3, b = 3·3, c = 22·7, V = 57·7%.

The head end is blunt, being thrice broader than deep, and not well set off from the rest of the body. The lips and the ampid are not clear. The stoma is slightly longer than broad measuring 18·464 μ by 16·156μ, being in the form of a cup. It consists of dorsal, middle and ventral ribs of moderate length. The dorsal tooth is pointed, not massive, being situated almost at the top. The base of the dorsal rib is obliquely turned inwards. The mid rib is in the form of a cross bar at the top. The ventral sector has six rows of denticles, the first two rows being situated at the top and bottom margins of the cross bar of the ventral rib. There are three submedian onchii in the ventral sector. The base of the ventral rib is swollen.

The oesophagus is muscular and the cardia is obscure. The rectum is oblique with a slit-like anus. The tail is ventrally bent. The gonads are paired,, opposed and are almost made invisible by the densely packed intestine. The entrance of the vulva is guarded by two cuticular plates. No male was recorded in the collection.

* Part of M.Sc. thesis approved by the Madras University.

Habitat : Soil near the roots of *Mimosa* plant.

Locality : Presidency College garden. No. 72.

Mononchus (*Mylonchulus*) *madrasi* sp. nov. is related to *Mononchus brachyuris* (Butschli 1873, Cobb 1917) in having two ovaries, the dorsal tooth being relatively small the amphid being very obscure but differs from it, in that the caudal glands are not saccate. The spinneret is not dorsally oblique but centrally placed. Further the tail is ventrally bent near its middle part. There is a cross bar in the ventral rib and there are three submedian onchii in the ventral sector.

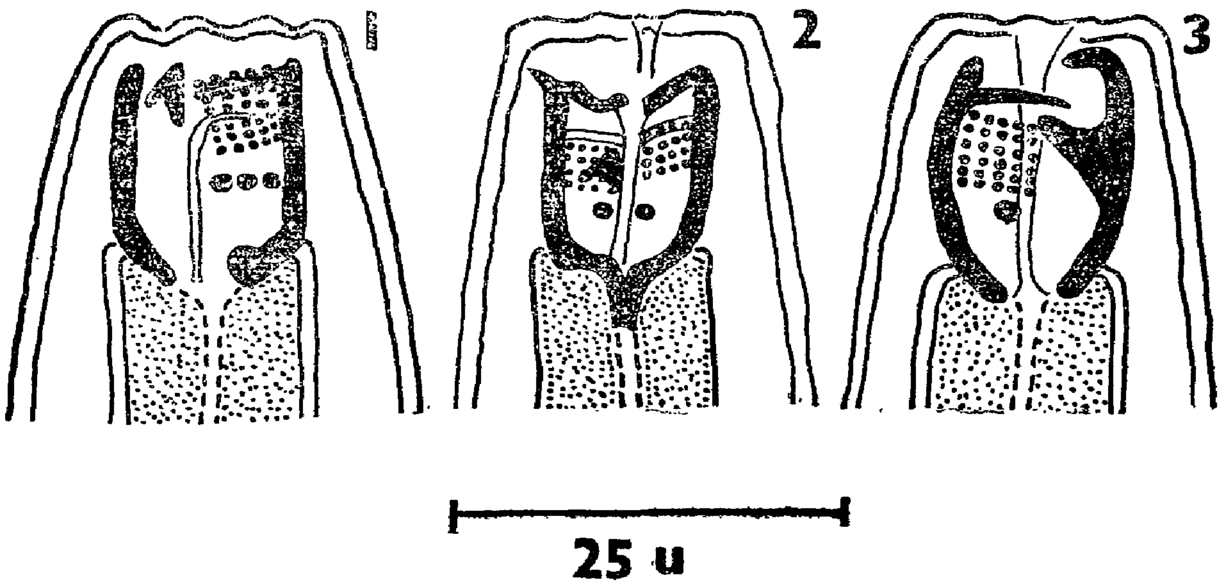


FIG. 1. *Mononchus* (*Mylonchulus*) *madrasi* sp. nov. Female Head end. FIG. 2. *Mononchus* (*Mylonchulus*) *subtenuis* var. *subventralis*. var. nov. Female Head end. FIG. 3. *Mononchus* (*Mylonchulus*) *obtusicaudatus* Cobb. Female Head end.

Holotype : 1 Female mounted, labelled and deposited in the departmental museum. Number Pck 43.

Paratypes : were not preserved

2. *Mononchulus* (*Mylonchulus*) *subtenuis* Cobb, var. *subventralis* var. nov. (Text-fig. 2.)

Total 49. Females 29. Juveniles 20.

Measurements. L = 0·99 mm. a = 19·8, b = 3·2., c = 22·4., V = 59·8%

The head end is blunt and flat, not well set off from the rest of the body, being 30 μ wide and 5 μ deep. The stoma is 20·772 μ long, 16·156 μ wide and wider at the mid region, with a dorsal, ventral sector being wider than the dorsal sector, with both

the sectors having cross bars. The dorsal and the ventral sector have each, four rows of denticles forming the rasp areas. The first row of the denticles in the ventral sector is situated on the cross bar itself. There are two submedian onchii one for each sector. The dorsal tooth is shifted subventrally towards the middle of the stoma.

The oesophagus is tubular and muscular, with a slightly expanded posterior part. The nerve ring is situated at the anterior third of the oesophagus. The cardia is not visible. The intestine is densely packed with granules and calls for no special mention. The anus is quite prominent The tail is ventrally bent as in other *Mononchus*. The nature of the gonads is not clear, being covered by the dense intestine. The vulva is post equatorial in position. No male was recorded in the collection.

Habitat & Locality } Same as in the case of the previous forms.

This new variety is related to *Mononchus* (*Mylonchulus*) *subtenuis* Cobb (1917) in its characters of the stoma, and in the other body characters but differs from it, in having four rows of rasps in each sector, the first row of the ventral sector, being situated on the ventral cross bar itself. Further the dorsal tooth is shifted subventrally towards the middle of the stoma.

Holotype: 1 Female mounted, labelled and deposited in the departmental museum. Number P. C. K. 44.

Paratypes were not preserved.

3. *Mononchus* (*Mylonchulus*) *obtusicaudatus* (Daday).
Cobb. (Text-fig. 3.)
Daday 1899. Cobb 1916.
Total 12 Females.
Measurements: L = 1·05 mm. a = 21., b = 33·6., c = 30., V = 55·64%

The body is moderately stout and cuticle is plain. The head is not well set off, being five times wider than deep and with amalgamated lips.

The stoma is globet shaped, being 23·08μ long and 16·156 μ wide. The dorsal tooth is very prominent. The ventral rib has a cross bar opposing the dorsal tooth. The "rasp" area consists of six rows of denticles with a submedian onchus below the "rasps".

The oesophagus is stout and muscular. It is 297μ long and 35 μ thick and envelops the base of the stoma. The cardia is flat. The intestine runs straight and is packed with fine granules. The rectum is almost transverse, while anus is slit-like. The anal diameter is approximately equal to the length of the tail. The latter is ventrically bent, with a blunt tip with two caudal glands and a spinneret. The gonad is single, anterior and with two cuticularised plates at the vulval opening. No male was recorded in the collection.

Habitat: Soil near the roots of *Canna* spp.

Locality: Marina garden plot No. 79.

Acknowledgements

The author expresses his grateful thanks to the late Prof. P. J. Sundara Rao for his guidance and to Prof. H. Enoch and to Prof. P. K. Menon for their interest in this work. He is also grateful to the Government of India for a Junior Research Scholarship during the course of the work.

REFERENCES

Bastian, C. H. (1865) Monograph on the *Anguillulidae* etc. *Trans. Linn. Soc. Lond.* 25: 73-184.

Bütschli, O. (1873) Beiträge Zur Kenntniss der freilebenden Nematoden. *Nova Actakgl. Leop.-Carol. Deutsch. Akad. Naturf.* 36: 1-144.

Cobb, N. A. (1916) Notes on new genera and species of Nematodes. 4. Subdivisions of *Mononchus. J. Parasitol.* 2: 195-196.

—— (1917) The *Mononchs* (*Mononchus* Bastian 1865), a genus of free living nematodes. *Soil Sci.* 1: 431-486.

—— (1918) Filter-bed nemas. Nematodes of the slow sand filter beds of American Cities etc. *Contrib. Sci. Nematol.* 7: 189-213.

—— (1920) One hundred new nemas. *Ibid.* 9: 217-343.

Goodey, T. (1942) Observations on *Mononchus tridentatus* and *M. brachyuris* and other species of the genus *Mononchus. J. Helmin,* 20: 9-24.

—— (1951) *Soil and Fresh-water Nematodes.* (*A Monograph.*) *Methuen,* Lond.

Kannan, S. (1960) Soil Nematodes from Madras City. *J. Zool. Soc. India* 12: 40-50.

Thorne, G. (1924) Utah nemas of the genus *Mononchus. Trans. Amer. Micr. Soc.* 43: 157-171.

J. Madras Univ., B, XXXI, No. 1, Pages 63-68, 1961.

Soil Nematodes of the Genera *Mononchus* and *Mononchulus**

BY

S. KANNAN

Department of Zoology, Presidency College, Madras

(Received for publication on 7-1-61)

ABSTRACT

The present paper describes five species of free living soil nematodes from Madras city. Of these two are new species, Viz: *Mononchus* (*Mononchus*) *neotumbridgensis* and *Mononchulus rotundicaudatus.*

1. *Mononchus* (*Mononchus*) *neotumbridgensis* sp. nov. Text-Fig. 1).

Total: 8 females.

Measurements: L = 0·836 mm., a = 14·3, b = 3·1, c = 14, V = ?

The cuticle is smooth. The head end is blunt, 13·84 μ wide, with amalgamated lips. It is separated by a shallow neck from the posterior part. The stoma is twice as long as broad, being 18·464μ in length and 9·232μ in width, with parallel sides and with capitate heads. Each side of the stoma has a pointed tooth at the top. There is a centrally placed tooth. There are no rasps.

The oesophagus is plain and cylindrical with the nerve ring surrounding it. The cardia seems to be absent. The rectum is transverse with a slit like anus. The tail has a large caudal papilla. The gonad is not visible due to dense nature of the brownish coloured intestine. No male was recorded in the collection.

Habitat: Soil near the roots of a fern spp. in a pot.

Locality: Green house, Botanical Gardens, Teynampet No. 66.

*Part of M.Sc. thesis approved by the University of Madras

This new species is closely related to *Monochus tumbridgensis* Bastian (1865), Cobb (1916) in the nature of the stoma, the centrally placed pharyngeal tooth, which is less than half the length of that of the species described by Bastian (1865). The two stomal ribs have each a tooth. Further, the head end is more rhomboidal in shape and the cephalic notches are deeper than in *Mononchus tumbridgensis* Bastian (1865).

Holotype: 1 Female mounted, labelled and deposited in the Departmental Museum, Number 35 P.C.K.

Paratypes: Mounted and labelled Number 35 (a) P.C.K.

2. *Mononchus* (*Prionchulus*) *muscorum* Bastian (Text-Figs. 2, 3 & 4).

Bastian 1865; Cobb 1916.

Total 18. Females 8, Juvenilles = 10.

Measurements: L = 2·0 — 2·5 mm. a = 26-31, b = 4-4·2, c = 14-15, V = ?

The body is short, 0·04 mm. wide with smooth cuticle. The head is not well set off. The lips are six in number and not prominent. The amphid is slit-like.

The stoma is globet shaped with well developed dorsal and ventral sectors and with a large dorsal tooth at the anterior third of the stoma, being opposed by longitudinal teeth on the ventral sector. The ventral rib is supported by a "leg" pushing into the oesophagial tissue. A transverse ridge encircles the stoma at the top of the dorsal tooth. There is also a similar ridge at the bottom of the dorsal tooth but faintly developed. Two additional submedian longitudinal ridges are also seen in the stoma. There is a foramin in the basal subdorsal plate.

The oesophagus is tubular and muscular with its anterior part extending up to nearly half the base of the stoma. The cardia is flat. The intestine is straight, and dense, ending in a slit like anus, the rectum being oblique. The tail is slightly greater than the anal body diameter, being convex dorsally, and bent ventrally with a blunt terminus, the caudal gland being absent. The gonad has a well developed anterior part and an attenuated posterior part. It was not possible to locate the vulva. No male was recorded in the collection.

Habitat: Soil near the roots of Mimosa tree.

Locality: Presidency College Garden No. 67.

3. *Mononchus* (*Prionchulus*) *muscurum* Bastian, var *macrolaimus* Cobb (Text-fig. 5).

Cobb 1917.

Total 4. Females 3. Juvenile 1.

Measurements: L = 2·1-2·4 mm. a = 26-30, b = 4-4% c = 14-15, V = 51-52%.

The cuticle is unstriated. The head is not well set off and the lips are not very distinct. The stoma measures 18·464 μ long and 11·54 μ wide. The head is 18·5 μ wide. The stoma is longer than broad, nearly one and half times its breadth.

The dorsal tooth is massive, while the ventral sector is thin with the teeth arranged longitudinally. A transverse ridge at the top of the dorsal tooth is prominent. The oesophagus is cylindrical, muscular and the cardia is very thin. The intestine is densely packed with granules, while the anus is slit like. The tail is curved ventrally with a gland like tissue. The anal diameter is half the length of the tail. The caudal gland and the spinneret are not clear. The ovaries are two, opposed and reflexed. The vulva is slightly post equatorial. No male was recorded in the collection.

Habitat & Locality } Same as in the case of the previous form.

4. *Mononchus* (*Iotonchus*) *cosimilis* (Text-fig. 6).
Cobb 1917.

Total 4. Females 2. Juveniles 2.

Measurements: L = 1·2 mm., a = 20, b = 3·9, c = 12·3, V = ?

The cuticle is plain with the head set off with 6 confluent lips, of which only the extreme ones are distinct. The head measures 20 μ in width. The stoma is well cuticularised, 30 μ long, 20 μ wide being one and half times as long as broad. The dorsal tooth is comparatively small, situated basally and opposed by two median onchii.

The oesophagus is simple, tubular and muscular measuring 310 μ by 30 μ. The cardia is not visible in this specimen. The intestine is dense. The tail is straight and digitate, with a caudal gland opening through a spinneret at the tip of the tail. The tail has five papillae, ventrally placed while there is a sixth one dorsally placed. The tail is nearly thrice the anal diameter. The

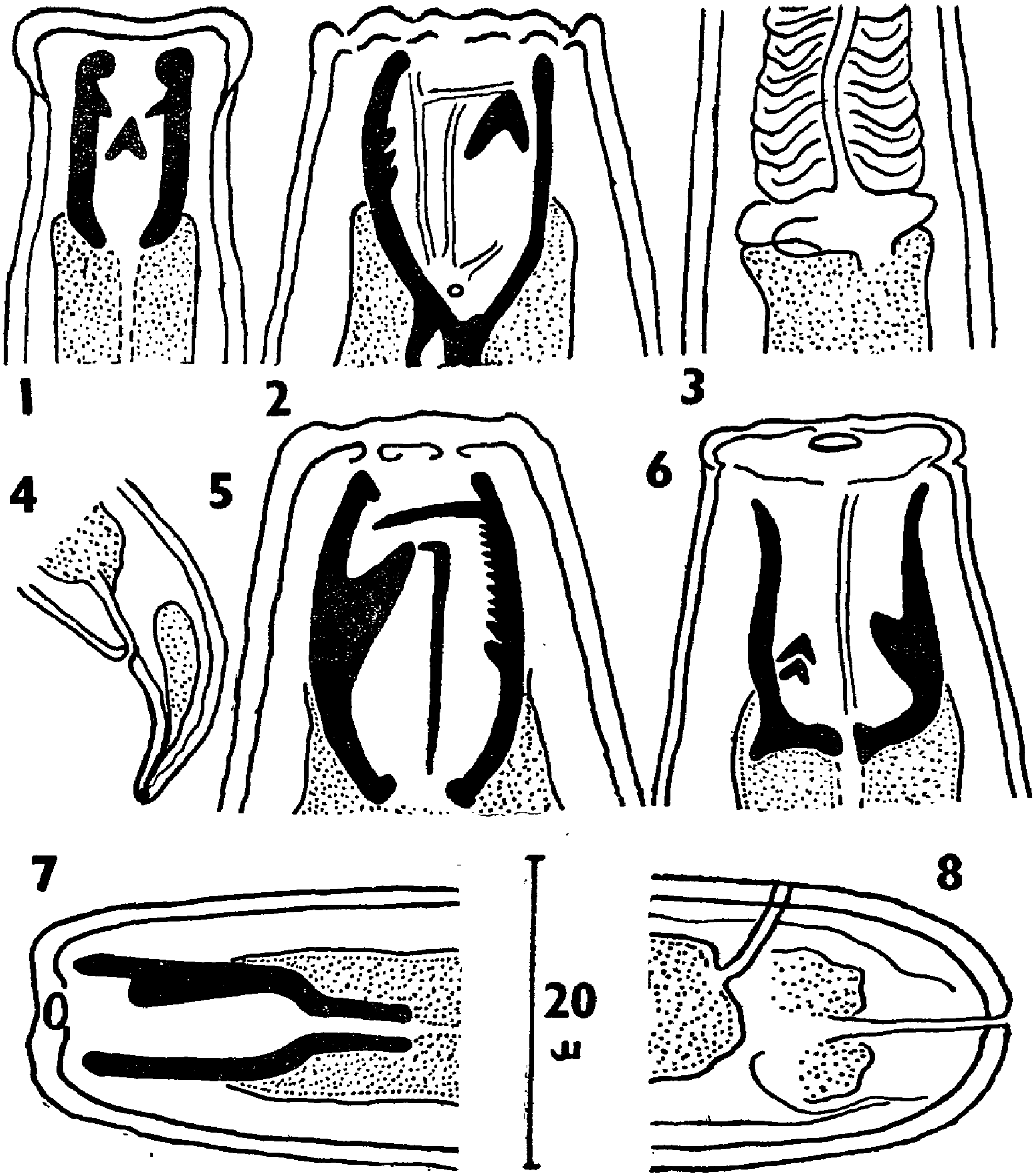


FIG. 1. *Mononchus* (*Mononchus*) *neotumbridgensis* sp. nov. Head end of the female. FIG. 2. *Mononchus* (*Prionchulus*) *muscorum* Bastian, Cobb. Head end of the female. FIG. 3. *Mononchus* (*Prionchulus*) *muscorum* Bastian, Cobb. Region of the oesophagus and intestine of the female. FIG. 4. *Mononchus* (*Prionchulus*) *muscorum* Bastian, Cobb. Tail end of the female. FIG. 5. *Mononchus* (*Prionchulus*) *muscorum* Bastian var. *macrolaimus* Cobb. Head end of the female. FIG. 6. *Mononchus* (*Iotonchus*) *Cosmilis* Cobb. Head end of the female. FIG. 7. *Mononchulus rotundicaudatus* sp. nov. Head end of the female. FIG. 8. *Mononchulus rotundicaudatus* sp. nov. Tail end of the female.

gonads are not very well developed and it was not possible to locate the vulva. No male was recorded in the collection.

Habitat & Locality } Same as in the case of the previous form.

5. *Mononchulus rotundicaudatus* sp. nov. (Text-Figs. 7 & 8).
Total 1 Female.

Measurements: L = 1·001 mm., a = 22·2, b = 4·0, c = 43·5, V = 54·7%.

The body is cylindrical while the cuticle is without straiae. The head is not well set off and without lips and bristles. The head end measures 16 μ in width.

The stoma is strongly cuticularised, deep, 20·7 μ long, 6·9 μ wide, with a simple dorsal sector. The ventral sector of the stoma has a big shield like tooth. No additional teeth or onchii are present. The oesophagus encircles the basal part of the stoma, at the anterior end, while the posterior end is slightly enlarged. The oesophagus is one fourth the total body length. The cardia is two-celled of which the dorsal cell is small and the ventral one is large. The intestine runs straight almost filling the body cavity while the anus is slit like. The tail is blunt, with a centrally placed spinneret. The caudal gland occupies ⅞th the length of the tail. The anal diameter is equal to the length of the tail.

The ovary is single, anterior, outstrected and reflexed. The vulva is a simple slit, post equatorial in position, being situated at 54·7% from the anterior end. The body is widest at the region of the ovary. The sides of the vulva are not cuticularised. The uterus is a simple sac. No male was recorded in the collection.

Habitat: Soil near the roots of *Canna* spp.
Locality: Marina garden plot Number 79.

Mononchulus rotundicaudatus sp. nov. is related to two closely allied species: *M. ventralis* Cobb (1918) and *M. nodicaudatus* (Daday 1901) Schneider 1937 (Syn. *Prismatolaimus nodicaudatus* Daday 1901) in its general characters of the body, gonad, stoma, cardia etc., but differs from them (the two above mentioned species according to Goodey (1951), being the same, since they are similar in size and in general characters), in that, the oesophagus is slightly shorter in length. Further, the length of the tail in *M. routundicaudatus* sp. nov. is tiwce that in *M. ventralis* Cobb (1918). The caudal gland opens out through

the spinneret which lies at the centre of the tail and not ventrally as in *M. ventralis* (Cobb 1918).

Holotype: The only female was mounted, labelled and deposited in the Departmental Museum, Number 32 P.C.K.

Acknowledgements

The author takes this opportunity to express his grateful thanks to the late Prof. P. J. Sundara Rao for suggesting the problem and guidance and to Prof. P. K. Menon for his suggestions. He is also thankful to the Government of India for the award of a Junior Research Scholarship during the tenure of this work.

REFERENCES

Bastian, C. H. (1865) Monograph on the Anguillulidae, etc., *Trans. Linn. Soc. Lond.* 25: 73-184.

Cobb, N. A. (1916) Notes on new genera and species of nematodes. 4. Subdivisions of *Mononchus. J. Parasitol.* 2: 195-196.

—— (1917) The *Mononchs.* (*Mononchus* Bastian 1865), a genus of free living nematode. *Soil Sci.* 1: 431-486.

—— (1918) Filter-bed Nemas. Nematodes of the slow sand filter-beds of American cities, etc. *Contrib. Sci. Nematol.* 7: 189-213.

—— (1920) One hundred new nemas. *Ibid.* 9: 217-343.

Goodey, T. (1942) Observations on *Mononchus tridentatus* and *M. brachyuris* and other species of the genus *Mononchus, J. Helminth.* 20: 9-24.

—— (1951) *Soil and Fresh water Nematodes.* (*A. Monograph*). Methuen. London.

Kannan, S. (1960) Soil nematodes from Madras City. *J. Zool. Soc. India.* 12: 40-50.

Schneider, W. (1937) Freilebende Nematoden der der Deutschen Limnologischen Sunda-expedition nach Sumatra, Java and Bali. *Arch. Hydrobiol.* (*Plankt*). 31: *Suppl.* 15. *Tropische Binnenggewasser.* 30-108.

Thorne, G. (1924) Utah nemas of the genus *Mononchus. Trans. Amer. micr. Soc.* 43: 157-171.

J. Madras Univ., B. XXXI, No. 1, Pages 69-74, 1961.

Nematodes of the Genus *Mononchus* from Madras City*

BY

S. Kannan,
Department of Zoology, Presidency College, Madras

(Received for publication on 7-1-1961)

ABSTRACT

A description of four species of *Mononchus* (Bastian 1865), a group of predatory soil nematodes is given in this paper. Of these, two are new varieties. All the four forms are recorded for the first time in Madras.

1. *Mononchus* (*Mylonchulus*) *montanus Thorne* (Text-fig. 1). Thorne, 1924

Total 46. Females 28., Juveniles 18.

Measurements. L = 0·974 mm., a = 16·2., b = 3·4., c = 25·2., V = 56·53% — 60%

Body not very long. Head end blunt, bearing six faintly seen lips. Head 23·08 μ wide, 4·616 μ deep. Amphid circular.

Stoma globet shaped, well cuticularised, measuring 20·77 μ by 16·16 μ. The dorsal sector has a large massive tooth situated midway between the pharynx. The ventral sector is quite wide, with six rows of denticles forming the "rasp" area, which is bounded by two transverse bars, one above and one below. There are three sub-median onchii below the "rasp" area.

The oesophagus is tubular, muscular, nine times longer than broad. The cardia is flat and not prominent. The intestine is densely packed with granules. The rectum is 0·2 mm. long and opens out by a slit-like anus.

The nature of the gonads is not clear due to dense structure of the intestine. The uterus bears a single ovate egg (0·15 mm. by

* Formed a partial fulfilment of M.Sc. thesis approved by the Madras University.

0·05 mm.). The vulva is post-equatorial in position, with two triangular plates at its entrance. The tail is convex dorsally, bent ventrally with a caudal gland. No male was recorded in the collection.

Habitat: Soil near the roots of mimosa tree.

Locality: Presidency College garden. No. 72.

2. *Mononchus* (*Mylonchulus*) *montanus* Thorne, var. *trionchulus* var. nov. (Text-fig. 2).

Total 48. Females 21. Juveniles 27.

Measurements: L = 1·17 mm., a = 23·4., b = 3·3., c = 15·05., V = 60-61%

The body is fairly long and stout. The head end is flat with indistinct lips. The stoma is well cuticularised, compressed slightly at the sides giving a leniar shape, being twice longer than broad. The dorsal sector is wide. The dorsal rib is plain, wide at the anterior and middle part and narrows down at the bottom. The dorsal tooth is not very large, situated midway between the stoma. It is not connected with the dorsal rib, but shifted away from the dorsal rib.

The ventral rib of the stoma is serrated with four teeth like ridges and with two cross bars projecting into the stoma. There are six rows of denticles in the ventral sector forming the "rasp" area, being bounded by the two transverse bars. There are three submedian onchii in the stoma, situated below the dorsal tooth. The dorsal sector has a single row of fine denticles.

The oesophagus is tubular and muscular. The intestine runs straight, opening out by a slit-like anus. The tail is, as in the other *Mononchs*. The caudal gland is trifolicate. The gonads are paired, opposed and reflexed with a slit-like vulva. No male was recorded in the collection.

Habitat & Locality } Same as in the case of the previous forms.

Mononchus (*Mylonchulus*) *montanus* Thorne var. *trionchulus* var. nov. is closely related to *M.* (*Mylonchulus*) *montanus* Thorne (1924) in its general body characters. The points of variations which differentiate this new variety are: the dorsal tooth is not very massive, neither does it form a part of the dorsal rib. On the contrary, it is a separate piece of moderate size and it is shifted

away from the dorsal rib. The dorsal rib has a single row of denticles. The ventral rib is characteristically serrated with teeth like ridges.

The characters similar to both of the above mentioned forms are: the ventral "rasp" area (denticles) is bounded by the transverse bars. There are three submedian onchii and six rows of denticles in both the forms.

Holotypes. 2 females mounted and deposited in the departmental museum. No. P.C.K. 42.

Paratypes: Did not stand preservation.

3. *Mononchus* (*Mylonchulus*) *sub-tenuis.* Cobb, (Text-fig. 3). Cobb, 1917.

Total 6 Females.

Measurements: L = 1·1-1·21 mm., a = 22., b = 3·36., c = 15·75., V = ?

Body fairly long with smooth cuticle. The head is 25·4 μ wide and is well set off by deep notches. Only four lips are faintly seen. The stoma is globet shaped, twice longer than broad with a mid rib dividing it into a dorsal and ventral sector. The tooth is fairly massive and centrally placed. There are two "rasp" areas, one for each sector, having six rows of denticles. There are two submedian onchii, one for each sector, placed in the middle of each sector. Each rib has a cross bar.

The oesophagus, intestine and anus call for no special mention. The tail is as in other *Mononchs.* The ovaries are paired, opposed and reflexed. The vulva could not be located. No male was recorded in the collection.

Habitat & Locality } Same as in the previous case.

4. *Mononchus* (*Mylonchulus*) *subtenuis* Cobb, var. *bionchus* var. nov. (Text-fig. 4).

Total 6. Females 4. Juveniles 2.

Measurements: L = 1·15-1·17 mm., a = 23., b = 3·2., c = 23., V = 58%

The body is fairly long with smooth cuticle. The head end is not well set off by notches and has four faintly seen lips. The head measures 15μ. The stoma is globet shaped and well cuticularised.

The tooth is not centrally placed, but shifted to the dorsal submedian position in the dorsal sector. The dorsal rib is plain and slightly swollen in the upper middle part. The ventral rib is serrated with four teeth like ridges. The ventral "rasp" area consists of five rows of denticles while the dorsal "rasp" area consists of six rows of denticles. There are two submedian onchii, one for

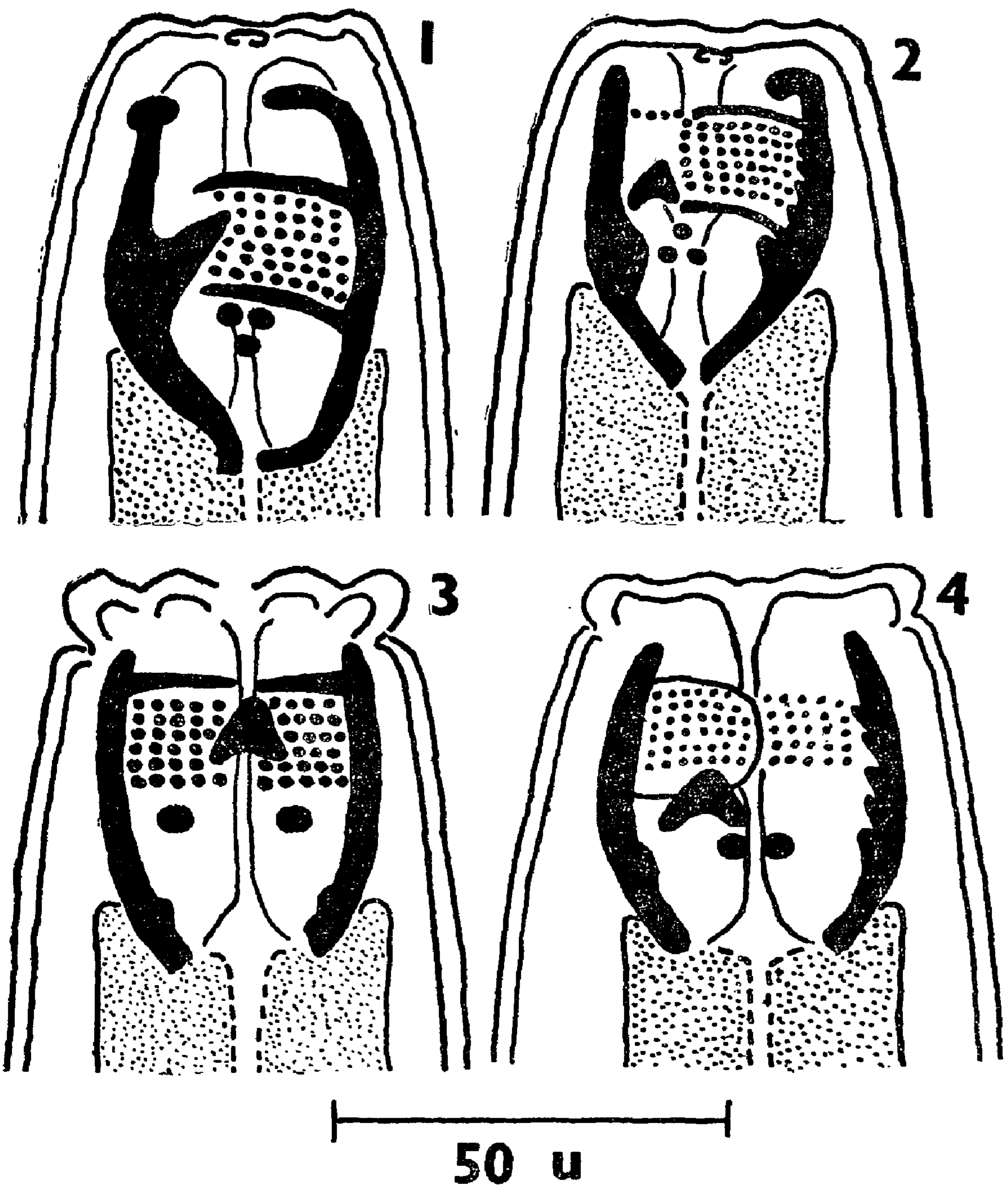


FIG. 1. *Mononchus* (*Mylonchulus*) *montanus* Thorne. Head end of the female. FIG. 2. *Mononchus* (*Mylonchulus*) *montanus* Thorne var. *trionchulus* var. nov. Head end of the female. FIG. 3. *Mononchus* (*Mylonchulus*) *subtenuis* Cobb. Head end of the female. FIG. 4. *Mononchus* (*Mylonchulus*) *subtenuis* Cobb var. *bionchus*. var. nov. Head end of the female.

each sector and these two onchii are not wide apart, but are very near the mid rib, near the centre of the stoma. The dorsal "rasp" area is encircled by an oval rib while the ventral cross bar is absent.

The alimentary canal calls for no special mention. The anal diameter is less than half the length of the tail. The gonads are paired, outstretched and reflexed. The vulva is post-equotorial in position. The tail is ventrally bent and with a caudal gland. No male was recorded in the collection.

Habitat & Locality } Same as in the case of the previous forms.

Mononchus (*Mylonchulus*) *subtenuis* Cobb var. *bionchus* var. nov. is closely related to *M. subtenuis* Cobb (1917), in the nature of the stoma with two rasp areas, and in its other features. The points of variations are: the ventral "rasp" area has five rows of denticles, while the dorsal rasp area has six rows. In *M. subtenuis* Cobb (1917) both the sectors have six rows of denticles in each. The tooth is not centrally placed, but shifted to the dorsal sector. Further the ventral rib is serrated with four teeth, and the ventral cross bar is absent. The dorsal "rasp" is bounded by an ovate rib.

Holotype : 1 Female mounted and deposited in the departmental museum. No. P.C.K. 41.

Paratypes: were not preserved.

Acknowledgements

The author takes this opportunity to convey his grateful thanks to the late Prof. P. J. Sundara Rao, for his guidance and to Prof. H. Enoch and Prof. P. K. Menon for their interest in the work. He is also grateful to the Government of India for the award of a Junior Research scholarship.

REFERENCES

Bastian, C. H. (1865) Monograph on the *Anguilluilidae*, etc. *Trans. Linn. Soc. Lond.* 25: 73–184.

Cobb, N A. (1916) Notes on new genera and species of nematodes. 4. Subdivisions of *Mononchus*. *J. Parasitol.* 2: 195–196.

—— (1917) The *Mononchs* (*Mononchus* Bastian 1865), a genus of free living nematodes. *Soil. Sci.* **1**: 431-486.

—— (1918) Filter-bed Nemas. Nematodes of the slow sand filter beds of American cities, etc. *Contrib. Sci. Nematol.* **7**: 189-213.

—— (1920) One hundred new nemas. *Ibid.* **9**: 2171343.

Goodey, T. (1942) Observations on *Mononchus tridentatus* and *M. brachyuris* and other species of the genus *Mononchus. J. Helminth.* **20**: 9-24.

—— (1951) *Soil and Freshwater Nematodes.* (*A monograph*) *Methuen.* London.

Kannan, S. (1960) Soil nematodes from Madras city. *J. Zool. Soc. India.* **12**: 40-50.

Thorne, G. (1924) Utah nemas of the genus *Mononchus. Trans Amer. micr. Soc.* **43**: 157-171.

J. Madras Univ., B. XXXI, No. 1, Pages 75-95, 1961.

On the Net of Conics with a Common Self-polar Triangle*

(PART I)

BY

B. R. SRINIVASAN

(Received for publication on 28-2-1961)

Introductory

The main idea in this thesis is that the invariant theory of two conics is essentially the same as the invariant theory of a fundamental triangle and two points in its plane. In fact, the whole thesis may be taken to be an explanation of this statement in its various aspects.

Any two conics have in general a common self-polar triangle and thus belong to the net of conics having this triangle for a common self-polar triangle. It can be further shown that any conic-invariant of the two conics is also necessarily a member of the net. Now the net contains three special members invariantly associated with it, namely, the three line-squares which are the sides of the self-polar triangle of the net. If we represent these three line-squares by three arbitrary points in a complex projective plane, not on the same line and a non-degenerate conic-locus of the net by an arbitrary point in general position with respect to the fundamental triangle formed by the representations of the line-squares, it follows from the general theorem of projectivity that a unique 1 — 1 correspondence is set up between the conics of the net and the points of the plane. Thus the conic-invariants of the two conics are represented precisely by the points which are invariantively associated with the fundamenntal triangle of the representation (as we call it) and the two points representing the two conics.

It now turns out that the theory of polar reciprocation among the conics of the net becomes the theory of quadratic involutions

* Madras University M.Sc. thesis, 1959.

with the fundamental triangle of the representation for the fundamental triangle of the involutions. The apolar theory of conics is reduced to the polar theory of the fundamental triangle by our representation.

Let S and S′ be the representations of the two conics, ABC being the fundamental triangle of the representation. The points which are invariantly associated with the triangle ABC and the point pair S, S′ and which arise naturally are as follows:

1. The pole of the line S, S′ with respect to the unique conic through A, B, C, S, S′. 4.3 says that this is the representation of the F-conic of the two conics.
2. The fourth intersection of conics through A, B, C touching SS′ at S and S′ respectively. By 4.6 this is the representation of the Ø — conic of the two conics.
3. The unique conjugate points of S and S′ with respect to the pencil of conics through S′ and S respectively and having ABC for the common self-polar triangle. These are by 4.11 the representations of the polar reciprocals of S and S′ with respect to S′ and S respectively.
4. The four fixed points of the quadratic involution with ABC for the fundamental triangle and S, S′ for corresponding points. 4.10 shows that these represent the four conics which reciprocate S and S′ into each other.
5. Pole of the line SS′ with respect to the triangle ABC. This *represents the F-conic of the unique pair of doubly apolar* conics in the pencil formed by S, S′; which is a combinant for the pencil. (4.43)
6. The two intersections of the polar conic of the pole of SS′ with SS′. These represent the unique pair of doubly apolar conics in the pencil formed by S and S′. (4.43) These are Hessian points of the three intersections of SS′ with the sides of the triangle ABC.
7. The pole of the unique conic Σ through A, B, C, S, S′, with respect to the triangle ABC. This represents the F-conic of the unique pair of doubly apolar conics in the range formed by S, S′; which is a combinant for the range (4.44).
8. The two intersections of the polar line of the pole of conic Σ with respect to ABC with Σ. These represent the unique pair of doubly apolar conics in the range formed by S and

S′. (4.44) These are the Hessian points of the triangle ABC on Σ.

9. The cubic-covariant points of the intersections of the sides of ABC with SS′ and also those of the triangle ABC on Σ. These represent respectively the three conics in the pencil and range detrmined by S and S′ which have double contact respectively with the combinants in 5 and 7. (4.48)

The thesis is divided into six chapters according to the subject matter. In Chapter I, we prove that the necessary and sufficient condition that a triangle is self-polar to a conic is that it should be left invairant by the four group of the triangle: and consequences.

In Chapter II, after a short account about the nature of conics with a common self polar triangle in section (1), we proceed in section (2), to define the F and Ø conics of two conics in a manner naturally suggested by the course of our arguments. We later [Chapter IV, (2)] prove from our definition that the F and Ø conics possess the property contained in the usual definitions. Section (3) of this chapter is devoted to a study of quadratic involutions which we pursue as for as we require for our purposes in later chapters. We close this chapter with a short section giving two classical examples of quadratic involutions which arise in the camplex Euclidian Geometry of the plane.

Chapter III is subdivided into five sections. In the first section we define apolarity of plane curves and prove just those results on apolar curves which we require for the development of the polar theory of the triangle in the next section. In section (2) we develop the polar theory of the triangle just to such an extent as we shall require in the apolar theory of conics to be developed in Chapter IV. Section (3) expresses the apolarity of the three-point and a three-line in terms of the polar theory. Section (4) discusses an important configuration of four triangles required later, while the next section indicates the connection of the configuration in (4) with the theory of a non-singular cubic and its inflexions.

In Chapter IV we come to the representation theory. It is this chapter which is devoted to the explanation of the main principle of this thesis. In (1) we discuss the representation and its immediate consequences. We then represent the F and Ø conics in (2) and use the representation to prove their harmonic locus

and harmonic envelope properties. We also give here ruler constructions for the representations of the F and Ø conics of two conics represented by two given pointts in the plane of the fundamental triangle ABC. The next section shows that the theory of polar reciprocation of conics of the net is essentially the theory of quadratic involutions of the fundamental triangle. We here obtain the representations of the polar reciprocals of a conic with respect to another given conic; the points representing them being given. Also the representations of the four conics reciprocating two given conics into each other are obtained; and the ruler constructions for these points. Section IV is devoted to show that the apolar theory of two conics is essentially the polar theory of the triangle. Section (5) is a short application of the representation theory of 'Harmonic Conics'.

Chapter V deals with the metrical specialisation of the representation by introducing an arbitrary conic through A, B, C, as the conic of 'parobolic points' and an arbitrary point of the plane not on this conic as the 'circular point' and consequences.

Chapter VI indicates the possibility of the dual representation in which conic-envelopes of the net are represented by points of the plane. After explaining the representation in (1) and noting the corresponding the metrical specialisation in (2) we proceed to an important particular case of the dual representation in (3); viz., the representation of a conic by its centre. The theorems in Chapters IV, V, VI now translate themselves readily into theorems on centres and centre-loci of conics and their relations with the centres of the conic invariants of two conics. The more important of these results are merely stated here.

It will be remarked that throughout the thesis we have appealed to pure geometric arguments with little or no use of coordinates.

Chapter I

The Four Group Associated with a Triangle

If I_1, I_2, I_3 denote three involuntary plane collineations with the vertices and opposite sides of a triangle ABC for their respective centres and axes (so that if P is any point in the plane $I_1(P)$ is the harmonic conjugate of P with respect to A and the intersection of AP and BC etc.) and I denotes the identity, then by

definition $I_1^2=I_2^2=I_3^2=I$. Also it may be easily verified directly from definition that $I_2I_3=I_1$, $I_3I_1=I_2$, $I_1I_2=I_3$. Hence I, I_1, I_2, I_3 form a group known as the four-group. Thus we have the result:

The three involuntary plane collineations which have their centres and axes for the vertices and opposite sides of a triangle together with the identity form the four group of the triangle.

Let now ABC be a self polar triangle of the conic S. Let any line through A cut S at X and Y and BC at P. Then (AP, XY) $= -1$. And hence I_1 interchanges X and Y. Thus S is invariant under I_1. So it is under I_2 and I_3 also. Thus S is left invariant by the four group of the triangle ABC. Conversely, if a conic S is left invariant by the four group of the triangle ABC, a point X on S is transformed into another point Y on S by I_1: so XY passes through A and if XY meets BC at P then (PA, XY) $= -1$. Hence A and P are conjugate points with respect to S. Since this is true for all points P on BC, it is the polar of A with respect to S. Similarly polars of B and C with respect to S are CA and AB respectively. Hence ABC is a self polar triangle for S. These are summed up in the following result:

1.1. The necessary and sufficient condition that a triangle is self polar for a conic is that the conic should be left invariant by the four group of the triangle.

Now it is well known that any two conics having distinct intersections have the diagonal point triangle of the quadrangle formed by the intersections for the common self-polar triangle. Hence by 1.1 these conics are left invariant by the four group of the common self-polar triangle. Since any conic-invariant of S and S′ is invariant under any collineation of the plane which leaves S and S′ invariant, it is clear that any conic-invariant of S and S′ is invariant under the four group of the common self polar triangle of S and S′. Hence it has the common self polar triangle of S and S′ for a self-polar triangle. Thus we have the fundamental result:

1.2. Any conic-invariant of two conics belongs to the net of conics which have the common self polar triangle of the two conics for a self polar triangle.

Thus the invariant theory of two conics is in a sense contained in the theory of net of conics having a common self-polar triangle.

The well-known conic-invariants of two conics like their F and Ø conics, the polar reciprocal of either with respect to the other are all therefore members of the net determined by the common self-polar triangle of the two conics. We shall return to these conic-invariants presently.

CHAPTER II

The Net of Conics having a Common Self-Polar Triangle

By the above, Conics having a given self polar triangle have three linear restrictions on their coefficients and hence belong to a linear system with two degrees of freedom, that is, a net.

Section (1)

Nature of the Net

The line-pairs of the net are those harmonically separating two of the sides of the common self polar triangle. The squared lines in the net are the three sides of the common self polar triangle. Any two connics of the net determine a pencil of conics which are all in the net. Pencil of conics having double contact contains a side of the self polar triangle squared. When considered from the dual point of view, the point pairs in the net are those harmonically separating two of the vertices of the self polar triangle. The three vertices squared are the point squares in the net. Any two conics in the net determine a range in the net and range of conics having double cotact contains a vertex squared of the self polar triangle.

The four group of the self polar triangle of the net transforms any point in the plane into the four vertices of a quadrangle having the self polar triangle for the diagonal point triangle and hence the pencil of conics in the net through a given point passes through all the vertices of the quadrangle generated by the point under the four group. In a similar mannner the four group generates the quadrilateral associated with the net from any given line.

Now consider any conic of the pencil in the net determined by a given point. The four group of the self polar triangle leaves this conic invariant and hence carries the tangent at the given point into the tangents at the corresponding points to the conic. Hence the conic is inscribed in the quadrilateral generated by the tangent at the given point under the collineations of the four

group, the points of contact being the vertices of the quadrangle generated by the given points under the four group. But quadrangles and quadrilaterales generated by the four group have the self polar triangle of the net for their diagonal point and line triangles. Hence we have:

2.1. The tangents at the vertices of a quadrangle to a conic circumscribing the quadrangle form a quadrilateral having the diagonal point-triangle of the quadrangle for its diagonal line triangle.

Now we are in a position to define the F and Ø conics of two conics by our approach.

Section (2)

F and Ø conics of two conics

Consider the points of contact of common tangents to two conics S and S′. The two quadrangles formed by them have the diagonal line triangle of the quadrilateral formed by the common tangents for their diagonal point triangle by 2.1. And this is the common self polar triangle of S and S′. Now consider the unique conic F through the four vertices of one of these quadrangles and one vertex of the other. This has the common self polar triangle of S and S′ for a self polar triangle. Hence it should be left invariant by the four group of this triangle. Now it is an immediate consequence from the harmonic property of the quadrangle that the four group of the diagonal point triangle of the quadrangle permutes the vertices of the quadrangle among themselves. It now follows that F has to pass through the other three vertices of the second quadrangle. Thus we have:

2.2. There is a unique conic through the eight points of contact of common tangents to two conics with them, which we define to be the F conic of the two conics.

As a corollary from the above arguments we get the following results also :

2.3. There is a conic through the eight vertices of two quadrangles which have a common diagonal point triangle.

A complete dualisation of our arguments above is the following two results :

2.4. There is a unique conic touching the eight tangents at the four intersections of two conics to them, which we define as the Ø conic of two conics.

2.5. There is a conic touching the eight sides of two quadrilaterals which have a common diagonal line triangle.

It is at once clear from their definitions that the F and Ø conics of two conics are necessarily conic-invariants of the two conics. It now follows from 1.2 (or directly from definition) that the F and Ø conics belong to the net of conics determined by the common self polar triangle of the two conics. We reserve the identification of the F and Ø conics thus defined to a later section.

Section (3)

Quadratic Involutions Associated with the Net

By a quadratic involution, we shall mean throughout this thesis, a plane Cremona involutary quadratic transformation of the *isogonal type* (as opposed to the inversion type) *

We recall that any point in the plane of the net determines a definite pencil in the net, namely the pencil of conics through the four vertices of the quadrangle generated by the given point under the four group of the common self polar triangle of the net. Now this pencil of conics in turn determines a unique quadratic involution which transforms any point in the plane into its conjugate with respect to the pencil. In particular this quadratic transformation takes the base point of the pencil into themselves. Thus we are led to :

2.6. Any point in the plane of the net determines a unique quadratic involution having the vertices of the common self polar triangle of the net for the fundamental points and having the vertices of the quadrangle generated by the given point under the four group of the self polar triangle for the fixed points.

That the transformation is involutary and 1 — 1 in general is obvious from its definition. The conjugate of any point on a side of the self polar triangle is the opposite vertex and hence the transform of a vertex is any point on the opposite side. Thus the vertices are exceptional points for the transformation. Again the transform of any line in general position with respect to the

* For purposes of this section one may consult with advantage page 46 to 49 of Semple and Roth: *Introduction to Algebraic Geometry,* as also page 50-51 of Hudson: *Cremona transformation.*

self polar triangle is the locus of its poles with respect to the pencil and hence is a conic circumscribed to the self polar triangle. These considerations show that the transformation is quadratic Cremona with the self polar triangle for the fundamental triangle. The proof is complete when we observe that the base points in the pencil are self conjugate with respect to the pencil and hence constitute the four fixed points of the transformation.

Now we proceed to prove the converse of the above proposition, viz., any quadratic involution is essentially the process of taking congugates with respect to the pencil of conics through its fixed points. We first prove:

2.7. Any quadratic involution permutes with the collineations of the four group of its fundamental triangle.

Let $P' = T(P)$, where T is a quadratic involution with the fundamental triangle ABC. Let the involuntary collineation I_1 centre A axes BC take P and P′ into Q and Q′ respectively. If AP and AP′ meet BC at X and X′ then (AX, PQ) is equal to $(AX', P'Q') = -1$. Since T takes any line through a fundamental point into another line through the same, AP goes over into AP′ under T. Again $T(A) = X'$ when A is considered as a point of AP. Also $T(X) = A$. Since T is essentially a $1-1$ transformation, it leaves the cross-ratios invariant. Hence if $T(Q) = Q''$ $(AX, PQ) = (X'A, P'Q'') = -1$. Hence Q′ and Q″ coincide. This means that $I_1(P') = Q' \mid = T(Q) = TI_1(P) = AI_1T^{-1}(P')$ It follows $I_1T = T\,I_1$ and two other similar results. 2.7 follows.

Suppose now that P is a fixed point of T. Then by 2.7 if G is any element of the four group from the fundamental triangle of T, then $TG(P) = GT(P) = G(P)$ since P is a fixed point. Hence $G(P)$ is a fixed point. Thus we have

2.8. The four fixed points of a quadratic involution are the four transforms of any one of them by the four group of the fundamental triangle and hence they form the four vertices of a quadrangle having the fundamental triangle for its diagonal point triangle.

By the above result the pencil of conics through the fixed points has the fundamental triangle for its self polar triangle. Now by 2·6 the process of taking conjugates with respect to this pencil is a quadratic involution with the same fixed points and fundamental points as T. Hence since the fixed points and fundamental points are enough to determine a quadratic Cremona

transformation uniquely, these two transformation are identical. Hence we have:

2.9. Any quadratic involution takes any point in the plane into its conjugate with respect to the pencil of conics through its fixed points, and the fundamental triangle of the involution is self polar with respect to the conics of the pencil.

Since a pencil of conics is uniquely determined by its common self polar triangle and a pair of conjugate points we have:

2.10. Any quadratic involution is uniquely determined if its fundamental points and a pair of corresponding points are given and hence afortiare, a fixed point and the three fundamental points determine uniquely a quadratic involution.

2.6 and 2.9 together show that quadratic involutions of the plane having the given fundamental triangle are precisely those associated with the net of conics having this triangle for their common self polar triangle. We now proceed to find ruler constructions for the correspondent of a given point under a quadratic involution specified as in 2.10.

Let ABC be the fundamental triangle of T and $S' = T(S'')$. It is required to construct T(P) given P. First construct the double lines of the involutions determined by the pairs AB, AC : AS″, AS′. Let them be the lines APQ, ARS. Similarly construct the double lines BPS, BQR of the involution determined by the pair, BC, BA : BS″, BS′ cutting APQ and ARS at the points P, Q, R, S. It is clear now that S″ and S′ are conjugate points for any conic of the pencil through PQRS. Hence the points PQRS are the four fixed points of T. To construct now T(X) given X construct the harmonic conjugates of AX and BX with respect to APQ, ARS : BPS, BQR. The intersection of these two lines is T(X). The proof is obvious when we note that X and T(X) are by construction conjugate points for the pencil of conics through PQRS.

In the case when T is specified by a double point S″ and S′ coincide and hence in the above construction APQ and ARS are the lines AS′ and the harmonic conjugate of AS″ with respect to AB and AS etc.

The above discussion gives the following corollary:

2.11. The necessary and sufficient condition for two given pairs of points to be corresponding pairs under the same quadra-

tic involution is that their joins to any fundamental point of the transformation and the fundamental lines through that fundamental point should belong to the same involution.

2.9 species the fixed points of the quadratic involution having a given fundamental triangle and a pair of corresponding points as the base points of the unique pencil of conics having the fundamental triangle for the self polar triangle and the pair of corresponding points for conjugate points. We proceed to consider an interesting alternate geometrical specification of the same.

We first observe that if $P' = T(P)$, the unique conic through A, B, C, P, P′ is the transform of the line PP′ and vice versa. Conversely, if a line PP′ in general position intersects its transform under T, i.e. a conic Σ through ABC at P and P′: then $P' = T(P)$ and $P = T(P')$. For, because P is an intersection of Σ and PP′ T(P) should be an intersection of PP′ and Σ. Hence either $T(P) = P$ or $T(P) = P'$. If $T(P) = P$, since T is essentially $1-1$ $T(P') = P'$. This means that P and P′ are fixed points of T, contradicting the assumption that the line PP′ is in general position with respect to T. The same argument shows that if we take a line through a fixed point P but not through any other fixed point, the transform of that line under T should touch it at P.

With these preliminaries, let us suppose that P is a fixed point of T. S, S′ being a pair of corresponding points. Then the transform of SP is a conic through A, B, C S′ touching SP at P. Similarly the transform of S′P is a conic through A, B, C, S touching S′P at P. Thus P is a point of intersection of the two loci L and L′ where L is the locus of points of contact of tangents drawn from S to the conics of the pencil ABCS′ and L′ that of points of contact of tangents drawn from S′ to the pencil ABCS. But these loci are cubic curves passing through the five points A, B, C, S, S′. (It is well known that the locus of the points of contact of tangents drawn from a given point to the conics of the pencil is a cubic curve touching the lines joining the given point to the base points at the base points, passing through the vertices of the self polar triangle of the pencil, touching the line joining the given point to its conjugate with respect to the pencil at the given point and passing through the conjugate point itself). And since these are cubic curves, they intersect in nine points. The remaining four points of intersection are therefore the fixed points of T. We have as a corollary the following result:

2.12. The cubic loci associated with the point S and the pencil of conics ABCS′ and the point S′ and the pencil of conics ABCS intersect at four other points which are the base points of the pencil of conics having ABC for the self polar triangle and S, S′ for conjugate points.

The following is an instructive alternate construction for T(P) given P, T being specified by its fundamental triangle ABC and a pair of corresponding points S and S′. Let A′, B′, C′, be the harmonic conjugates of A, B, C with respect to S and S′ on the conic Σ through ABCSS′. We note that A′, B′, C′, can be constructed by ruler without constructing Σ. Because T preserves cross-ratio, T(A′) T(B′), T(C′) are harmonic conjugates of the intresections of BC, CA, AB with SS′ with respect to S and S′. Hence T(A′), T(B′), T(C′) can be constructed by ruler. Let Σ′ be the conic through ABCA′P. Then T(Σ′) is a line L′ through T(A′) and T(P) such that the cross-ratio of the points X″=T(A′) and intersections of BC, CA, AB with L′ is equal to the cross-ratio P(A′ABC) on Σ′. To construct L′ it is enough a line AX′ intersecting BC at X′ such that A(X″X′CB)=P(A′ABC). Then X″X′ is the line L′ etc., T(P) is the point of concurrence of L′M′N′.

When P is a fixed point of the transformation, we see from the above construction that L′ passes through P and because T(L′)=Σ′, Σ′ touches L′ at P. Thus P is a point of contact of a tangent from X″ to a conic of the pencil ABCA′ and so on. Hence the three cubic loci associated with X″ and the pencil ABCA′, Y″ and ABCB′, Z″ and ABCC′ should have four common points other than ABC, which are the four fixed points of T.

Section (4)

Two examples of quadratic involution

When the circular points at infinity are corresponding pairs, the pencil of conics through the fixed points is the pencil of the rectangular hyperbolas through the in and ex-centres of the fundamental triangle ABC. Two points are correspondents under the quadratic involution when their joins to A′, B′, C are separated harmonically by the bisectors of the angles A, B, C, viz. they are isogonal conjugates for the triangle ABC.

The other example is an example for a dual quadratic involution. Here the mid-point and the point at ∞ on the sides of the fundamental triangle ABC are the double points of the funda-

mental involutions on the sides. Two lines are correspondents if their intersection with the sides of ABC are reflexions with respective mid-points. The fundamental range of conics is the range of parabolas inscribed in the medial triangle of ABC.

Chapter III

The Polar and Apolar Theory of Triangles in a Projective Plane

Section (1)

*Apolarity of Plane Curves**

The point locus S represented symbolically by the point equation $S=a_x{}^m=b_x{}^m=0$, and the line envelope given by the line equation $\Sigma=l_\alpha{}^n=l_\beta{}^n=0$ are said to be apolar if $a_x{}^{m-n}a_\alpha{}^n=0$ $(m \geqslant n)$.

Thus apolarity is equal to the identical vanishing of a covariant linear in the coefficients of the point to one curve and the line equation of the other.

When the class of the envelope is the same as the degree of the locus, i.e., when $m=n$ in the above the condition for apolarity is $a_\alpha{}^n=0$, i.e., the vanishing of the lineo-linear invariant of the locus and the envelope.

A particularly interesting case is when the locus is an n-line and the envelope an *n-point.* Let $S=a_x{}^{(1)}a_x{}^{(2)}a_x{}^{(3)}\ldots a_x{}^{(n)}=0$ be the n-line which is apolar to Σ. Then $a_\alpha{}^{(1)}a_\alpha{}^{(2)}a_\alpha{}^{(3)}\ldots a_\alpha{}^{(n)}=0$ where $\Sigma=l_\alpha{}^n=l_{\alpha(1)}l_{\alpha(2)}\ldots\ldots l_{\alpha(n)}=0$. Hence the apolarity condition becomes.

3.1. $\Sigma\, a_{\alpha(1)}{}^{(\gamma_1)}a_{\alpha(2)}{}^{(\gamma_2)}\ldots\ldots a_{\alpha(n)}{}^{(\gamma_n)}=0$ where the sum runs over the factorial n permutations $r_1, r_2,\ldots r_n$, of the first n natural numbers.

The following is an immediate consequence of the definition of apolarity.

3.2. The necessary and sufficient condition that a locus of the nth degree is apolar to a point taken n times is that the point should lie on the locus.

3.1. at once gives the following two results.

* See Grace and Young: *Algebra of Invariants.*

3.3. Any n-line is apolar to an n-point situated entirely on one of its n-lines.

3.4. Any n-line is apolar to an n-point consisting of an arbitrary point and an intersection of two of its n-lines taken $n-1$ times.

Let now an n-point be situated entirely on a line, say the join of (y) and (z). Any point on the line is of the form $(\lambda_1 y+\lambda_2 z)$. If $\lambda^{(\gamma)}$, $\gamma=1,\ldots n$ are the parameters of the n points, then the n-point is given by $\Sigma=\overset{n}{\underset{1}{\pi}} l\,(\lambda_1{}^{(\gamma)}y+\lambda_2{}^{(\gamma)}z)=0$. i.e. $\Sigma=\overset{n}{\underset{1}{\pi}}(\lambda_1{}^{(\gamma)}l_y+\lambda_2{}^{(\gamma)}\,l_z)=0$. If now $S=a_x{}^n=0$ is apolar to Σ then $\overset{n}{\underset{1}{\pi}}(\lambda_1{}^{(\gamma)}a_y+\lambda_2{}^{(\gamma)}a_z)=0$. Putting now $a_y=A_1$, $a_z=A_2$ so that $A_1\lambda_1{}^{(\gamma)}+A_2\lambda_2{}^{(\gamma)}=A_\lambda{}^{(\gamma)}$ the relation becomes $A_{\lambda}{}_{(1)}A_{\lambda}{}_{(2)}\ldots A_{\lambda}{}_{(n)}=0$. But this is precisely the condition that the n points $\lambda^{(\gamma)}$, $\gamma=1\ldots n$ should be in *binary* apolarity with $A_\lambda{}^n=0$, i.e., with $(a_y\lambda_1+a_z\lambda_2)^n=0$ and the latter equation gives precisely the intersections of $a_x{}^n=0$ with the join of (y) and (z). Hence we have:

3.5. A necessary and sufficient condition for an n-point situated entirely on a line to be apolar to a locus of the nth degree is that the n-points on the line and the n intersections of that line with the locus should be in binary apolarity.

Section (2)

The Polar Theory of the Triangle

The results of the previous section will now be applied to a three point and a three line; i.e. the apolar theory of two triangles in a plane will now be made the basis for the polar theory of the triangle. We define

3.6. The first polar of a point P with respect to a triangle ABC is the locus of points Q such that the three-point PQQ is apolar to the triangle ABC considered as a three-line. The locus of a conic through A, B, C called the polar conic of P with respect to the triangle.

To prove this we first observe that the apolarity condition 3.1 involves *linearly* any line of the n-line or any point of the n-point. Also from 3.4 PAA is apolar to the three-line ABC. 3.6 follows immediately.

3.7. The second polar of P is defined as the locus of points Q such that the three-point PPQ is apolar to the three-line ABC: This is the polar line of P w.r. to ABC.

That the locus is a line is clear from 3.1.

The following result is an immediate consequence of 3.6 and 3.7.

3.8. If the polar line of P passes through Q, the polar conic of Q passes through P and vice versa.

Let now the harmonic conjugate of AP w.r. to AB, AC cut BC at X. Let PX cut AB and AC at Y and Z respectively. Consider now points P, Q such that the triad P, Q, X is binary apolarity to the triad X, Y, Z. Then to any P there is a unique Q and the relation between P and Q is symmetric. Hence P and Q generate an involution on the line XYZ. Z, Y are corresponding members of the involution since a binary form of odd order is apolar to itself. Also X is a double point of this involution for the triad XXX is apolar to XYZ. Hence the other double point is P. Therefore PPX is in binary apolarity with XYZ and hence by 3·5 the three-point PPX is apolar to the three-line ABC. This shows that the polar line of P w.r. to ABC passes through X. Similarly it passes through the intersection of harmonic conjugates of BP w.r. to BA, BC; CP w.r. to CA, CB; with CA and AB respectively. Thus we have:

3.9. If X, Y, Z are the harmonic conjugates of intersections of PA, PB, PC with BC, CA, AB w.r. to B, C; C, A, A, B respectively, then X, Y, Z are collinear and the line XYZ is the polar line of P w.r. to ABC.

Now let us use 3.9 to find the polar line of a point P which is infinitesimally near to A along a definite line through A. The points Y and Z coincide with A. But the point X is the harmonic conjugate with respect B and C of the intersection of BC with the definite line through A. Hence polar line of P with respect to ABC is AX. Hence we have

3.10. The polar line of a point P infinitesimally near to a vertex of a fundamental triangle on a definite line through that vertex is the harmonic conjugate of that line with respect to the sides through that vertex.

If now P is any point in the plane of the triangle and Q the point infinitesimally near to A on the harmonic connjugate of AP with respect to AB and AC, then by 3.10 polar line of Q pases through P. Hence by 3.8 polar conic of P passes through Q.i.e. Polar conic of P touches the harmonic conjugate of AP w.r. to AB and AC, etc. Hence we have

3.11. If AX, BY, CZ are the harmonic conjugates of AP, BP, CP with respect to the sides taken two at a time of ABC, there is a unique conic touching AX, BY, CZ at A, B, C respectively and this is the polar conic of P w.r. to ABC.

If P lies on BC, the lines BY and CZ coincide with BC. Since the polar conic of P should touch BC at B and C BC is a part of it. Since it should touch AX, AX is the other part. Hence we have

3.12. When P lies on a side of a triangle ABC, its polar conic breaks up into the line BC and the harmonic conjugate of AP w.r. to AB and AC. In particular, polar conic of a vertex of the triangle with respect to itself is the degenerate conic consisting of the sides through that vertex.

As a particular case of 3·9 we have that the polar line of a point on a side of a triangle with respect to itself is that side. The polar line of a vertex as such however is undefined, since any point in the plane together with a vertex squared is apolar to the triangle as a three-line.

Let now the polar line of P and the polar conic of P w.r. to ABC intersect at a point X. We assert that PX is a tangent at X to the polar conic. For, if it is not, let X and Y be the distinct intersections of PX with the polar conic of P. Then the triads PXX, PPX, PYY are apolar to the three-line ABC. Now points Q, Q′ on PX such that PQQ′ is apolar to the three-line ABC evidently form an involution with X and Y as double points. Hence if P′ is the harmonic conjugate w.r. to X and Y (this is distinct from X and Y since X and Y are distinct) of P then the triad PPP′ is apolar to ABC. Thus we have two distinct triads PPP′ and PPX apolar to ABC. Hence the polar line of P is PX. Hence PPP is apolar to ABC and therefore P, X, Y are three distinct intersections of the polar conic and polar line of P. This means that PX is a part of and hence is a tangent to polar conic of P. Thus in all cases the tangents at the points of intersection of the

polar conic and the polar line of a point P passes through P. Thus we have

3.13. The polar line of a point with respect to a triangle is the polar line of the point w.r. to the polar conic of the point.

Let now the polar line of the point P with respect to ABC cut BC, CA, AB at X, Y, Z so that by 3.9 P and X separate harmonically the points of intersection of PX with AB, AC. Now apply any quadratic involution T with the fundamental points A, B, C to the configuration. T(PX) is a conic through A, B, C, T(P) and T(X). Since T preserves cross-ratios B, C T(P), T(X) is a harmonic tetrad on this conic and further T(X) = A. Hence the line joining A and T(P) is the harmonic conjugate of the tangent AX′ at A to this conic w.r. to AB, AC. Now the polar line of P w.r. to ABC passes through X; hence it is transformed into a conic through A, B, C, touching AX′ at A. Similarly the transform of XYZ touches BY′ and CZ′ at B, C respectively. A glance at 3.11 now shows at once that T(XYZ) is the polar conic of T(P) w.r. to the triangle ABC. Thus the quadratic involution T takes the polar line of a point P into the polar conic T(P). If now Q is any point on the polar conic of P by 3.8 polar line of Q passes through P. Hence the polar conic of T(Q) passes through T(P). Hence again by 3.8 polar line T(P) passes through T(Q). Hence we have

3.14. Any quadratic involution with the fundamental triangle ABC transforms the polar line of a point w.r. to ABC into the polar conic of its transform and vice versa.

If now the pole of the conic through A, B, C w.r. to the triangle ABC lies on a line then consider the unique quadratic involution which has the intersections of this line with the conic for corresponding points and ABC for the fundamental triangle. It takes the line into the conic and vice verse. Now by 3.14 since the pole of the conic lies on the line the transform of the pole of the conic which is the pole of the line lies on the transform of the line, i.e., the conic. A similar argument proves the converse also. Hence we have

3.14a. If the pole of a conic through A, B, C w.r. to the triangle ABC lies on a line, the pole of the line w.r. to the same triangle lies on the conic and vice versa.

Section (3)

Apolarity of three-point with three-line

Now we can use the polar theory to specify when a three-point is apolar to a given triangle ABC. Let P_1, P_2, P_3 be the three-point apolar to the three line ABC. Consider pairs P, Q on P_2P_3 such that P_1PQ is apolar to the three-line ABC. Then P and Q form an involution of which P_2, P_3 are corresponding members. The double points of the involution are evidently the intersections of the polar conic of P_1 w.r. to ABC with P_2P_3. Hence P_2 and P_3 are conjugate points w.r. to the polar conic of P_1. Conversely if P_2 and P_3 are conjugate w.r. to the polar conic of P_1 then P_2 and P_3 are corresponding members of involution P, Q on P_2P_3 such that the triad P_1PQ is apolar to the triangle ABC. Hence the triad $P_1P_2P_3$ is apolar to the triangle ABC. And since apolarity is a symmetrical relation we have

3.15. If a triad $P_1P_2P_3$ is apolar to the triangle ABC then any two points of the triad are conjugate points w.r. to the polar conic of the third w.r. to ABC, and conversely, if one pair of a triad are conjugate w.r. to the polar conic of the third w.r. to a triangle, then every pair of the triad possess this property and the triad is apolar to the triangle.

Let $P_1P_2P_3$ be a triad of points apolar to the three-line ABC. Let the pencil of conics through A, B, C which have P_2, P_3 for conjugate points intersect at a fourth point $\emptyset_{23}$. Then by 3.15 polar conic of P_1 w.r. to ABC belongs to this pencil: i.e., polar conic of P_1 passes through $\emptyset_{23}$. Hence by 3.8 the polar line of $\emptyset_{23}$ passes through P_1. Thus we have:

3.16. The locus of a point which moves such that the triad formed by it and two other given points is apolar to a given triangle as a three-line, is the polar line w.r. to the triangle of the fourth base point of the pencil of conics through the vertices of the triangle, having the two given points for conjugate points.

Section (4)

A Configuration formed by Four Triangles

Consider now the points of intersection P_2, P_3 of the polar line and the polar conic of P_1 w.r. to the triangle ABC. Then by 3.13 polar conic of P_1 has P_1 and P_2P_3 for pole and polar. Now projecting P_2, P_3 into the circular points at infinity and denoting pro-

jections by small letters, P_1 is the centre of the circle abc and because the line at ∞ is the polar line of p_1 w.r. to the triangle abc, ap_1 meets bc at its midpoint. Hence ap_1 is perpendicular to bc. Hence in the original figure if X is the point of intersection of AP_1 and BC then $X(P_2P_3, P_1B) = -1$. Thus P_2 and P_3 are conjugate points w.r. to the degenerate conic P_1A, BC: similarly w.r. to the degenerate conic P_1B, CA and hence w.r. to the pencil of conics through P_1, A, B, C. Now consider the conic of pencil which touches P_1P_3 at P_1: then polar of P_3 w.r. to this conic is P_1P_2. But both the polar conic and polar line of P_3 w.r. to ABC pass through P_1 by 3.8. And hence polar conic of P_3 touches P_1P_3 at P_1 by 3.13. Thus P_1P_2 is the polar of P_3 w.r. to the polar conic of P_3. Hence P_1P_2 is the polar line of P_3 w.r. to ABC by 3.13. Thus polar line of P_3 passes through P_2. Similarly polar line of P_2 passes through P_3. Hence by 3.8 P_2 is the other intersection of the polar line and the polar conic of P_3. Thus in the triangle $P_1P_2P_3$ the polar line and the polar conic of any one vertex intersect at the other two vertices.

Hence the triads $P_2P_2P_3$ and $P_3P_3P_2$ are both apolar to the three line ABC. It now follows that the triad consisting of any point on P_2P_3 and the points P_2, P_3 is apolar to the three line ABC. Hence by 3.5 this triad is in binary apolarity with the three intersections of P_2P_3 with the sides of the triangle ABC. Therefore P_2 and P_3 are the Hessian points of the three intersections.

Again under the projection effected above abc is an equilateral triangle and the polar of the point at ∞ on bc is the line ap_1. Hence the polar of the points of intersections X′ of BC and P_2P_3 is AP_1. Thus AP_1 and AX′ separate both the pairs AP_2, AP_3 : AB, AC harmonically. This means that the pairs AB, AC, AP_2, AP_3 : AP_1, AP_1 are corresponding members of the same involution. This with two other similar results gives by 2.11 the result that the unique quadratic involution with ABC for fundamental triangle and P_1 for a fixed point has P_2, P_3 for corresponding points.

Now since abc is an equilateral triangle and P_1 its circumcentre, ap_1 meets the line at infinity at a point which is the harmonic conjugate of the point at infinity on bc w.r. to the circular points at infinity. Hence by 3.9, the polar line of 'a' w.r. to the triangle $p_1p_2p_3$ passes through the intersection of bc with p_2p_3. Hence the polar line of A w.r. to the triangle $P_1P_2P_3$ passes through the intersection of BC with P_2P_3. Since the relation of the triangle $P_1P_2P_3$ with ABC is symmetric w.r. to the vertices of the first

triangle, it follows that BC is the polar line of A w.r. to the triangle $P_1P_2P_3$. Hence by 3.8 B and C are the points of intersection of the polar line and the polar conic of A w.r. to $P_1P_2P_3$ and so on. Thus the relation the triangle ABC and $P_1P_2P_3$ is symmetric w.r. to their vertices and the two triangles themselves.

Again the limiting points of the coaxeal system of circles through b, c : c, a : a, b are situated two by two on the three lines ap_1, bp_1, cp_1 and they lie three by three on two circles centre P_1 forming two equilateral triangles with their sides parallel to those of abc. Hence any one of these triangles is in the same relation with the triangle $p_1p_2p_3$ as the triangle abc. The vertices of these two triangles are the six points of concurrence of lines like ap_1, bp_2, cp_3. Again the sides of these triangles pass through the points of intersection like bc, p_2p_3. Thus the vertices and sides of the two triangles are precisely the six centres and the six axes of perspective of the two sextible perspective triangles abc, $p_1p_2p_3$. And the symmetry of the whole figure, any two of the four triangles are in the same relation as any other two. Further when abc is real the two triangles formed by limiting points of coaxeal circles through ab, be, ca are conjugate imaginary triangles and a side and opposite vertex of $p_1p_2p_3$ are real. The results of this discussion can be summed up in 3.17.

3.17. *The triangle formed by a point and the two intersections of the polar line and the polar conic of the point w.r. to another triangle is in sextible perspective with this triangle, the centres and axes of perspective forming the vertices and sides of two other triangles. The relation between any two of the four triangles is symmetric w.r. to the two triangles and w.r. to the vertices of either triangle: and is the same for any two of the four triangles. Any two triangles are in sextible perspective with the centres and axes of pespective being the vertices and sides of the other two. Any triangle of the four is self polar w.r. to any other triangle of the four: i.e. the polar lines of vertices are opposite sides, the polar conics are conics touching the two sides through the vertex at the other two vertices. The triad consisting of any point on a side of a triangle and the two vertices on that side is apolar to any triangle of the four considered as a three line. Any two vertices of a triangle are the Hessian points of its three intersections with the sides of any other triangle. Any two vertices of a triangle are correspondents in the quadratic involutions having the third vertex for the fixed point and with any other triangle of the*

four either (i) for the fundamental triangle or (ii) for the triangle formed by the other three fixed points. The system of the twelve lines formed by the four triangle intersect in the same nine points,the join of any two of which passes through a third of them. Of the four triangles one is real, one has one real vertex and the other two vertices conjugate imaginaries, while the other two triangles are purely imaginary, they being conjugate imaginary triangles.

Section (5)

*Connection with the Theory of Inflexions of a Non-singular Cubic**

The configuration of four triangles discussed in the previous section occurs in the theory of inflexions of a non-singular cubic. Any non-singular cubic can be written in the canonical form $f = x^3 + y^3 + z^3 + 3mxyz = 0$. Its Hessian H is given by $H = x^3$ $y^3 + z^3 + 3m'xyz = 0$, where $m' = -(4+m^3)/3m^2$. Hence the pencil formed by f and H is of the form $\lambda(x^3+y^3+z^3) - 3\mu xyz = 0$. Now the condition that a cubic should degenerate into a triad of lines is that it should be the same as its Hessian. This gives the parameters μ/λ of the triad of lines of the pencil to be 1, α, α^2 ∞ where α is the cube root of unity. Accordingly we get the four triangles of the pencil, viz., $x=0$, $y=0$, $z=0$; $x+y+z=0$, $x+y\alpha^2+z\alpha=0$, $x+y\alpha+z\alpha^2=0$: $x+y+z\alpha=0$, $x+y\alpha+z=0$, $x\alpha+y+z=0$: $x+y+z\alpha^2=0$, $x+y\alpha^2+z=0$, $x\alpha^2+y+z=0$. Also it is known that H intersects f at its points of inflexion. Thus the nine points common to these four triangles are the points of inflexion of the cubics of the standard pencil determined by them. It is now easy Algebra to verify that the four triangles form the configuration described in 3.17.

* For an account of the theory of inflexions of a plane cubic one may consult page 41 and 42 of Semple and Roth: *Introduction to Algebraic Geometry* as also articles 173-74, 217 of Solmon: *Higher Plane Curves*. But in neither the sextible perspectivity of the four triangles is mentioned as also some other properties described in 3.17.

J. Madras Univ., B, XXXI, No. 1, Pages 97-107, 1961.

On Some Permutable Processes in Semi-groups

BY

N. Sankaran,
Department of Mathematics,
University of Madras

(Received for publication on 27-2-1961)

ABSTRACT

A study of a class of asymmetric uniform spaces defined by omitting from the definition of uniform spaces [1] the axiom of symmetry, was made by Krishnan [2]. The present paper concerns itself with semi-groups which are endowed with asymmetric uniform structures compatible with their law of composition and natural order. We show that the processes of group embedding, uniform completing, symmetrisation of the asymmetric uniformity and formation of direct product of a family of semi-groups that are naturally totally ordered are permutable.

1. Preliminaries : By a semi-group (S, +) is meant a commutative, associative, cancellable binary system with an identity element which we denote by 0. The semi-group S is said to be divisible if for every element x in S, and non-zero integer n we can find an element y in S such that

$$y + y + \ldots\ldots n \text{ times} = x.$$

We can introduce an ordering relation $\leqslant$ in S (called the natural order S) thus : $a \leqslant b$ in S if and only if there exists an element c in S such that $b = a + c$. That this is a partial ordering relation on S and is compatible with the semi-group operation + is immediate. This ordering is strict if and only if $a + b = 0$ implies $a = 0$ and $b = 0$. It is linear if and only if for given a, b either $a = b + c$, or $b = a + d$, where c, d are in S, holds. The semi-group is said to be naturally totally ordered if the natural order on S is a total order.

An ordered semi-group S is said to be weakly divisible if for any non-zero element x in S and any non-zero positive integer n,

Numbers in brackets refer to references at the end.

there exists an element y in S such that $n.y \leqslant x$. In the sequel by a weakly divisible semi-group we mean a semi-group which is weakly divisible with reference to its natural order. We assume that I, a non-null initial subset of $S - 0$, has no first element. Consequently we are concerned only with non-discrete uniform semi-groups.

By a uniform semi-group (S, +, U) we mean a semi-group with an asymmetric uniform structure which is compatible with the operation + in the following sense:

$(x, y) \varepsilon U_i$, $U_i \varepsilon U$ if and only if $(x + z, y + z) \varepsilon U_i$ for each z in S.

A semi-group (S, +) is uniformisable in its natural order if $U = \{U_i\}$ is given by

$$U_i = \{ (x, y) \mid y < x + i, \; i \text{ in } I\}$$

where '<' is the natural order of S and I is an initial set in $S - 0$. It is easy to see that this is an asymmetric uniformity for S which is compatible with the law of composition '+' of S.

The following result is easily obtained: Any divisible semi-group (S, +) which is naturally totally ordered is uniformisable in its natural order. A more general result (Theorem 1) is also true. This theorem characterises naturally totally ordered semi-groups which are uniformisable in their natural order.

THEOREM 1. *A naturally totally ordered semi-group* (S, +) *is uniformisable if and only if it is weakly divisible.*

PROOF. We notice that weak divisibility is essential for verifying the following axiom of a uniform space: Given a surrounding U_i, there exists a U_j such that the relational product $U_j \, o \, U_j \subset U_i$ is true. It suffices to prove that weak divisibility implies and is implied by the above statement. Suppose S is weakly divisible. Then for each i in S we can get a j in S such that $2j < i$. Then $U_j o \, U_j \subset U_i$ since

$$(x, z) \; \varepsilon \, U_j \text{ and } (z, y) \, \varepsilon U_j \rightarrow (x, y) \, \varepsilon U_i.$$

The converse follows by retracing the above steps.

In what follows by a uniform semi-group we mean a naturally totally ordered semi-group which is uniformisable in its natural

order. It is easy to verify that such a semi-group is dense in its natural order.

By a $\{I, <\}$ sequence of S, we mean a collection $\{x_a\}$ of elements of S indexed by $I (= S - 0)$. It is a Cauchy sequence when for each i in I we can find a non-null initial subset $A(i)$ of I (a in $A(i)$, b in I, $b < a$ imply b is in $A(i)$) suuch that for $a, a' \varepsilon A(i)$, $(x_a, x_{a'}) \varepsilon U_i$ where U_i denotes the surrounding of the diagonal. A $\{I, <\}$ sequence $\{x_a\}$ of S is said to converge (coconverge) to x if for each $i \varepsilon I$, a non-null initial subset $A(i)$ of I can be found such that when $a \varepsilon A(i)$, $(x_a, x) \varepsilon U_i$ ($(x, x_a) \varepsilon U_i$).

The space S* with the asymmetric uniform structure is said to be complete if each Cauchy sequence of S* converges and coconverges to a common element in S*.

Let (S, **U**) be any asymmetric uniform space. Let **C** denote the class of all Cauchy sequences of S with the following uniformity.

For X, Y ε **C**, $(X, Y) \varepsilon \bar{U}_i$ for i in I if and only if there exists a nonnull initial subset $A(i)$ of I such that for a, a' in $A(i)$, $(x_a, y_{a'}) \varepsilon U_i$ where $\{x_a\} = X$, $\{y_{a'}\} = Y$. We say that two sequences are equivalent if and only if they belong to $\bar{U}_i$ for each $i \varepsilon I$. Let E denote this equivalence. We represent **C**/E by S* and the quotient uniformity by $\mathbf{U}^* = \{\bar{U}_i\}$.

LEMMA 1. (THEOREM 1 of [2]. *If* (S, **U**) *is an asymmetric uniform space then there exists a space* (S*,**U***) *which is complete and a unimorphism f of* S *into* S* *such that f*(S) *is dense in* S*. *This* S* *is unique upto isomorphism and minimal among the completions of* S.

2. Topological Embeddng: We shall now state and prove the following:

THEOREM 2. If (S, **U**) *is a uniform semi-group then it can be embedded as a dense subsemi-group of a complete semi-group* (S*, **U***).

PROOF. Let S* denote the equivalent classes of Cauchy I-sequences of S (that is S* = **C**/E). In S* introduce addition as follows:

For X, Y in S*, $X + Y = \{x_a + y_a\}$ where $\{x_a\} = X$, $\{y_a\} = Y$. It is easy to see that this addition is unique upto equi-

valent Cauchy I-sequences. Therefore $+$ is well defined on S*. This operation is seen to be commutative and associative without difficulty while cancellation law is verified thus:

If $X+Y=X+Z$ (equality in the sense of equivalence) then $(X+Y,\ X+Z)$ and $(X+Z,\ X+Y)$ are in $\overline{U}_i$ for each $i \varepsilon I$ and this implies the existence of a non-null initial subset $A(i)$ of I such that for $a,\ a' \varepsilon A(i)$,[1]

$$(x_a+y_a,\ x_{a'}+z_{a'}) \text{ and } (x_a+z_a,\ x_{a'}+y_{a'}) \ \varepsilon\ U_i.$$

Now X being a Cauchy I-sequence, given i, we can find a non-null initial subset $B(i)$ of I such that for $a,\ a' \varepsilon B(i)$, $(x_a,\ x_{a'})$ and $(x_{a'},\ x_a) \varepsilon U_i$ so that from $(x_a+y_a,\ x_{a'}+z_{a'})$ and $(x_{a'}+z_{a'},\ x_a+z_a)$ in U_i we get $(x_a+y_a,\ x_a+z_{a'}) \varepsilon U_i o U_i$, that is $(y_a,\ z_{a'}) \varepsilon U_i o U_i \subset U_j$ where j and i are related as $2i<j$. This being true for each j (because for each j we can choose an $i \varepsilon I$ such that $2i<j$) we see that Y, Z are equivalent. Therefore cancellation law it true in S*. Thus S* is a semi-group. It is dense and the completeness follows from lemma 1.

We now introduce an ordering relation in S* and show that the natural order of S* coincides with this ordering.

DEFINITION 1. We say that $X \leqslant Y$ if and only if (i) for each $i \varepsilon I$, there exists a non-null initial subset $A(i)$ of I such that for j in $A(i)$, $x_j<y_j+i$, (or equivalently), (ii) for each $i \varepsilon I$, there exists a non-null initial subset $A(i)$ of I such that for j, k in $A(i)$, $x_j<y_k+i$.

It is easy to see that the above ordering is reflexive, transitive and strict. In order to show that this ordering coincides with the natural order of S*, we prove the following two lemmas.

LEMMA 2. *If* φ *is a mapping of* I *to* I *such that* $\varphi(i)=i'<i$, *and* $X=\{x_i\}$ *is a Cauchy* I-*sequence of* S, *then* $X'=\{x'_i\}$ *where* $x'_i=x_{i'}$ *is also a Cauchy* I-*sequence and* X' *is equivalent to* X.

PROOF. Since X is a Cauchy I-sequence we can find for each i a non-null initial subset $A(i)$ of I such that for $j,\ k \varepsilon A(i)$, $(x_j,\ x_k) \varepsilon U_i$. Now for $j,\ k \varepsilon A(i)$ if $\varphi(j)=j'$ and

1. Since the intersection of a finite number of non-null initial subsets of I is a non-null initial subset of I, we can take a, a', etc., from a suitable non-null initial subset.

$\varphi(k) = k'$ then $(x_{j'}, x_{k'}) \varepsilon U_i$ since $j', k' \varepsilon A(i)$. Thus X′ is a Cauchy sequence. By setting $x_{i'} = x'_i$, etc., we get X′ as a Cauchy I-sequence of S.

Obviously $(x_j, x_{k'}) \varepsilon U_i$ and $(x_{k'}, x_j) \varepsilon U_i$ for each i when $j, k' \varepsilon A(i)$ and this implies that X and X′ are equivalent.

LEMMA 3. *If* X, Y *are two Cauchy* I*-sequences such that* $X < Y$ *then there exists a Cauchy* I*-sequence* Z *such that* $X+Z=Y$.

PROOF. Let us choose subsequences X′ and Y′ from X and Y as in the above lemma with φ having the following meaning: $\varphi(i) = i'$ is an element of the non-null initial subset $A(i)$ of I corresponding to i. Then $X' < Y' \rightarrow$ for each i, there exists a non-null initial subset $A(i)$ of I such that for $j \varepsilon A(i)$ $x'_j < y'_j + i$ where $x'_j = x_{j'}$, and $y'_j = y_{j'}$ $\rightarrow x'_j + z'_j = y'_j + i$, since S is naturally totally ordered.

The set of z'_j will form a Cauchy I-sequence, for, given i choose j such that $2j < i$ and for j, let $A(j)$ be the non-null initial subset of I. Let $k, l \varepsilon A(j)$. If $k' = \varphi(k)$ and $l' = \varphi(l)$ then $(x'_k, x'_l) \varepsilon U_j$, $(y'_k, y'_l) \varepsilon U_j$. We have $(y'_k + j, y'_l + j) \varepsilon U_j$ and this implies that $(x'_k + z'_k, x'_l + z'_l) \varepsilon U_j$. Further we have $(x'_k + z'_l, x'_k + z'_l) \varepsilon U_j$. Combining these two we have $(x'_k + z'_k, x'_k + z'_l) \varepsilon U_j \circ U_j \subset U_i$. That is $(z'_k, z'_l) \varepsilon U_i$. This being true for each i we see that the set of z'_i forms a Cauchy I-sequence. From the choice of z'_i it follows trivially that $(X + Z, Y) \varepsilon \overline{U}_i$ for each i and this proves that $X + Z$ and Y are equivalent.

That the ordering introduced above is reflexive and transitive is easy to prove while strictness of the order is proved from the fact that $X+Y=0 \rightarrow X=0, Y=0$.

We observe that $X+Y=0 \rightarrow$ For each i, there exists a non-null initial subset $A(i)$ of I such that whenever $a \varepsilon A(i)$, $(x_a + y_a, 0) \varepsilon U_i$ and $(0, x_a + y_a) \varepsilon U_i$

$\rightarrow x_a, y_a < x_a + y_a < 0 + i$,

and $0 < x_a + y_a + i$

$\rightarrow x_a < i$, $y_a < i$ for each i and a suitable a

$\rightarrow (x_a, 0) \varepsilon U_i$ and $(0, x_a) \varepsilon U_i$

and $(y_a, 0) \varepsilon U_i$ and $(0, y_a) \varepsilon U_i$

$\rightarrow X = 0, Y = 0$.

We notice that S^* is naturally totally ordered if S is naturally totally ordered.

We are justified in using I as the indexing set for the uniformity of S^* because for any element of S^* we can find an image of an element of S under the unimorphism of lemma 1, which is less than the chosen element. In other words for any given sequence $X \neq 0$ in S^*, we can find a repeated sequence less than X. We prove this fact in the following:

LEMMA 4. *If Z is a positive sequence in S^*, then Z is greater than a repeated sequence.*

PROOF. $Z \not\leqq 0$ implies that there exists an i such that for every initial set there is a $z'_j = z_{j'} \nless i$, (i.e.) $z_{j'} \geqslant i$. If the initial set consists of elements less than j, then choosing a fixed $j' < j$ with the above property, we get $z_{j'} \geqslant i$. The set $\{z'_j\}$ forms a subsequence equivalent to Z. This subsequence is easily seen to be $\geqslant \{i\}$, the repeated sequence of i.

We now show that the intrinsic uniform structure **V** of S^* and the completion uniform structure **U*** are equivalent. The intrinsic uniform structure $\mathbf{V} = \{V_i\}$ is given by

$V_j = \{(Y, Z) \mid Z < Y + J$ where J is the repeated sequence of $j\}$.

Given a $i \varepsilon I$, choose a j such that $2j < i$.

$(X, Y) \varepsilon V_j \rightarrow Y < X + J.$

$\rightarrow$ For each k, we can find non-null initial subset $A(k)$ of I such that for every $a \varepsilon A(k)$, $y_a < x_a + j + k$. Since k is arbitrary, by putting $k = j$ we get $y_a < x_a + 2j < x_a + i$

$\rightarrow (X, Y) \varepsilon \bar{U}_i$.

Thus we see that for each $\bar{U}_i$ we can find a V_j such that $V_j \subset \bar{U}_i$

For the other implication, given i, $(X, Y) \varepsilon \bar{U}_i \rightarrow$ For i there exists a non-null initial subset $A(i)$ of I such that for every $a \varepsilon A(i)$, $(x_a, y_a) \varepsilon U_i$

$\rightarrow y_a < x_a + i$ for all $a \varepsilon A(j)$. This being true we have by the definition of order among Cauchy I-sequences $Y < X + I$, where I is the Cauchy sequence of i.

$\rightarrow (X, Y) \varepsilon V_i$.

Thus for a given V_i we can find a $\bar{U}_i$ such that $\bar{U}_i \subset V_i$

Thus we have proved the following:

THEOREM 3. *The natural order of* S *can be extended to the natural order of the uniform completion* S*; *for* S* *the intrinsic uniformity and the completion uniformity are equivalent.*

3. GROUP EMBEDDING: It is well known that S can be embedded in a minimal group G unique upto isomorphism. In case S is naturally totally ordered, every element of G is in S or in $-S$ where $-S$ denotes the collection of elements $\{-x, x \varepsilon S\}$ such that $x + (-x) = 0$.

When S is a semi-group with uniform structure **U** we introduce in G the uniformity $\mathbf{W} = \{W_i\}$ as follows:

$(x, y) \varepsilon W_i \longleftrightarrow (x+z, y+z) \varepsilon U_i$ where $z \varepsilon S$ such that $x+z$, $y+z \varepsilon S$.

The group embedding of S* will be denoted by $\bar{G}$ and is seen to have the uniformity $\bar{\mathbf{V}}$ given by

$(X, Y) \varepsilon \bar{V}_i \longleftrightarrow$ there exists a sequence Z (in S*) such that

$(X+Z, Y+Z) \varepsilon \bar{U}_i$ where Z is such that $X+Z, Y+Z \varepsilon S^*$.

We see without difficulty the truth of the following:

LEMMA 5. *The completeness of* S* *will imply the completeness of the group* $\bar{G}$.

It is easy to prove the following:

LEMMA 6. *If* G *is the group embedding of* S *and* G* *is the uniform completion of* G, *then a Cauchy sequence not equivalent to the zero (sequence) of* G* *consists ultimately of elements that are all positive or all negative.*

PROOF. Assume that the sequence Z is not equivalent to zero and $Z \nless 0$. This means that Z is not ultimately negative. Then for each i we can choose a $i' \leqslant i$ such that $z'_i = z_{i'}$ is positive; therefore the susequence $\{z'_i\} = Z'$ of Z which is equivalent to Z, is positive. Therefore, $Z \lesseqgtr 0 \to Z$ is equivalent to $Z' > 0 \to Z > 0$.

Now we state and prove the following:

THEOREM 4. *If* G *is the group embedding of* S *with the associated uniform structure and* G* *is its uniform completion*

then G^* is unimorphic with the group embedding $\bar{G}$ of the uniformly complete semi-group S^*.

PROOF. We shall denote by $\tilde{S}$ the positive part of G^*. S^* is obviously the positive part of $\bar{G}$. There is a group isomorphism φ between $\bar{G}$ and G^* which is defined as follows: Let φ be the mapping that takes the elements of S (that is repeated sequences from $\tilde{S}$) to the corresponding elements in S^*. If X is any arbitrary sequence in $\tilde{S}$ then $\varphi(X)$ will be the sequence formed by the collection of elements of X which are elements of S. That is $\varphi(X)$ contains a residual set of X consisting of elements of S only. It is a matter of easy verification that φ takes equivalent sequences to equivalent sequences. Put $\varphi(X) = X'$. Then if X', Y' correspond respectively to X, Y then $X' + Y'$ corresponds to $X + Y$. Thus φ is addition preserving. If $X^* \varepsilon S^*$, then X^* consists of positive elements of G and hence belongs to $\tilde{S}$. Thus φ is seen to be onto. It is easily seen to be $(1 - 1)$ upto equivalence. Also if $X > 0$ then $\varphi(X) > \varphi(0) = 0$. Thus φ is order preserving. Since the uniformities are introduced by the order in both the cases S^* and $\tilde{S}$ and therefore also in $\bar{G}$ and G^*, it is evident that φ, which preserves the order, is a unimorphism. Hence the theorem.

We have to bear in mind that though G and G^* are two unimorphic groups with asymmetric uniform structures with respect to which they are complete, they are topologically only complete semi-groups and not complete groups. For, even though addition is a uniformly continuous operation, subtraction is not. If we now symmetrise the uniform structure in both the cases we get two unimorphic topological groups, because under the symmetric uniformity subtraction is a continuous operation and we have the following theorem of [2] which shows that symmetrisation and uniform completion are permutable processes.

Let (S, U) denote a space with asymmetric uniformity and (S, V) the symmetric associate of (S, U). Let (S^*, U^*) and (S^*, V^*) denote the topological completions of (S, U) and (S, V) res-

pectively. Let $(\bar{S}, \bar{V})$ be the symmetric associate of (S^*, U^*). Then we have

THEOREM 5. (THEOREM 2 of [2]). *The space* $(\bar{S}, \bar{V})$ *is unimorphic with the space* (S^*, V^*).

We can now deduce the following:

THEOREM 6. *In a naturally totally ordered weakly divisible semi-group, symmetrisation of the intrinsic asymmetric uniformity, uniform completion and group embedding are permutable processes.*

We can extend the above theorem to the direct product of a family of such semi-groups and get the following theorem:

THEOREM 7. *Let* $\{S_\lambda\}$ *be a family of naturally totally ordered weakly divisible semi-groups. If* P, U, G, S *denote respectively the formation of direct products, uniform completion, group embedding, and symmetrisation of asymmetric uniformity then we get unimorphic groups in whatever manner we apply the processes* P, U, G, S *to the family* $\{S_\lambda\}$.

For proving this theorem we require the following two lemmas.

LEMMA 7. *The group embedding* G *of the direct product* S *of an arbitrary family* $\{S_\lambda\}$ *of semi-groups and the direct product* G′ *of group embeddings* $\{G'_\lambda\}$ *of* $\{S_\lambda\}$ *are isomorphic.*

PROOF. In the group G each element g is a pair of sequences of elements from various S_λ and in G′ every element is a sequence of pairs of elements from S. It is a matter of easy verification that if a pair of sequences $(\{g_\lambda\}, \{g'_\lambda\})$ representing an element g of G corresponds to the sequence of pairs $\{(g_\lambda, g'_\lambda)\}$ corresponding to an element g' of G′, then any pair of sequences equivalent to g goes over to a sequence of pairs equivalent to g' and conversely. Thus the mapping between G and G′ is $1-1$ both ways. Since addition is componentwise in both G and G′, it is easy to see that G and G′ are isomorphic.

We now prove:

LEMMA 8. *The uniform completion of the direct product and the direct product of uniform completion (of a family of commutative weakly divisible semi-groups) are unimorphic.*

PROOF. Let S^*_λ denote the uniform completion of S_λ. Then S_λ is dense in S^*_λ. To show that $P_\lambda S^*_\lambda$ is the same as $(P_\lambda S_\lambda)^*$ it is enough to show that $P_\lambda S_\lambda$ is dense in $P_\lambda S^*_\lambda$ which is complete because it is the direct product of complete spaces.

To show that $P_\lambda S_\lambda$ is dense in $P_\lambda S^*_\lambda$, let us take a point $x \varepsilon P_\lambda S^*_\lambda$ and a neighbourhood of $U^{\lambda_1, \lambda_2, \ldots \lambda_n}_{i_1, i_2, \ldots i_n}(x)$ and show that

$$U^{\lambda_1 \lambda_2 \ldots \lambda_n}_{i_1, i_2, \ldots i_n}(x) \cap P_\lambda S_\lambda \neq \emptyset \text{ Now } U^{i_1, i_2, \ldots i_n}_{\lambda_1 \lambda_2 \ldots \lambda_n}(x)$$

$= P_k U^{\lambda_k}_{i_k}(x_{\lambda_k})$ where x_{λ_k} is the λ_k-th coordinate of x and $k = 1, 2 \ldots . n$ and at other places whole spaces are taken as neighbourhoods. Further x_{λ_k} is in $S^*_{\lambda_k}$ and S_{λ_k} is dense in $S^*_{\lambda_k}$ which implies that $U^{\lambda_k}_{i_k}(x_{\lambda_k}) \cap S_\lambda \neq \emptyset$ and this is true for $\lambda_1, \lambda_2, \ldots \lambda_n$. Therefore there exists y_{λ_k} in $U^{\lambda_k}(x_\lambda) \cap S_\lambda$ for $\lambda_1, \lambda_2, \ldots, \lambda_n$ and for other places we can take any arbitrary point from the component space. This shows that

$(y_1, y_2 \ldots y_{\lambda_1} \; y_{\lambda_2}, y_{\lambda_3}, \ldots y_{\lambda_n} \ldots)$ is in $U^{\lambda_1, \lambda_2 \ldots \lambda_n}_{i_1, i_2, \ldots i_n}(x)$ and in $P_\lambda S_\lambda$. Therefore $P_\lambda S_\lambda$ is dense in $P_\lambda S^*_\lambda$.

LEMMA 9: (THEOREM 3 of [2]). *If* $(S_i, (U_j^i))$ $i \varepsilon I, j \varepsilon J(i)$ *be a family of uniform spaces and* $(S^*_i, (V^i_j))$ *are the complete associates, then the complete associate of the direct product of non-complete asymmetric uniform spaces is unimorphic to the direct product of the complete associates of the individual spaces.*

Now the proof of the theorem 6 is evident from the theorems 3 and 4 and lemmas 7, 8 and 9.

We give a very simple example to illustrate our theorem.

Example: Consider the semi-group of non-negative rationals under the usual ordering. The uniform completion of this semi-group under the uniformity given by the natural order gives the semi-group of reals with the one-sided (left) uniform structure and its group completion gives the set of all reals with the order uniformity under which only addition is a uniformly continuous operation. If we now symmetrise this uniform structure then we get the usual topological group of all reals. This is the same as the symmetrised uniform structure of the topological completion of the set of all rationals which is the group completion of the semi-group with which we started.

The author takes the opportunity to thank Prof. V. S. Krishnan for his helpful suggestions and advice in the preparation of this paper.

REFERENCES

1. Bourbaki, N. .. *Topologie Generale.* Hermann & Co., Paris, 1951. Livre III.

2. Krishnan, V. S. .. Sur la completion des espaces a structure uniforme non-symmetrique. (In press).